ACADEMY OF MUSIC AND DRAMA

Towards the creation of a twenty-four notes based system.

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Abstract

Keywords: *Quarter tones; Micro-tonality; Tonal system; Semiotic analysis of music; Mathematical formalization; twenty-four notes system; Microtonal guitar; Quarter tone jazz; Syntactic structures*

This research consists in the proposal of a method to organize a twenty four notes based tonal system.
It starts considering music structure comparing it to syntax and grammar of the verbal language, and it develops through a mathematical formalization and a related geometric representation of tonal functions. It is proposed a method to create scales and harmonies with quarter tones.
The material got from the method is lately used into a practical music context: related audio files are attached to the work.
This research wants to be useful for musicians who wants to approach quarter tones in tonal context independently from extra European cultures.
The music has been developed on the classical guitar, thus there is a brief explanation of the instrumental issues that concern playing quarter tones on it, and some related theoretic considerations.

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Una parola senza suono
è come una girandola senza cielo.
Muta.

(E.Degani)

For the music, which has given me so much.
To my parents: Dana and Valter.

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Notes on the text

Media files.
In this text there are often mentioned certain media files. They are part of the work as long as this text and therefore they are attached to it on the H.S.M. web portal. The media files are divided into two sections: to one side they belong audio examples of the scales here proposed (there are also attached three small videos), to the other one they belong files come from the process I am still involved in to produce a recording session with Diana Torto and Michele Rabbia (See chapter 6). They are some examples took from rehearsals which are meant to correlate the theoretical work here exposed.

The audio examples of scales are structured in the following way:

- first it is exposed the scale from the root the octave in the ascendent movement.
- then it is proposed again the scale with the root as a bass note.
- finally it is proposed a brief improvisation to give the colour of the mode.

Furthermore it has been used the symbol (♫) to indicate in the text when there is an audio file related to the string of text marked with this sign.

It is also used the symbol (⋆) to indicate the video file related.

All the media files are listed before the chapter 1.

Text generation.
The present work has been produced using **LaTeX.** [1]
Latex is a free software typesetting system which is very suitable for producing scientific and mathematical documents of any type, of high typographical quality. The system is also suitable for producing all sorts of other documents, from simple letters to complete books. LATEX uses TEX as its formatting engine, and is the standard de facto in the academic word, particularly in the scientific context. Latex has a great number of packages to manage different types of specific needs, such as drawing geometrical figures, defines automata or other types of specific graphs, and so on. We have used this software because in this thesis

[1] see `https://www.latex-project.org/`

there is the need to express mathematical concepts, and to draw geometrical schemas.

Scores writing.
The musical scores present in the thesis has been produced using **Lilypond**. [2]
Lilypond is a free software released under the terms of the GNU General Public License, for music engraving. It has a professional level, and it is integrated with LaTeX.
It has been used instead of other programs such as Finale or Sibelius, because it allows us to deal with a *quarter tone notation*, which is fundamental to be used to write scores in this thesis.

[2] see `http://lilypond.org/`

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Chapter 1

Preface

Before I start the writing, I want to apologize with the reader if sometimes the text looks clumsy and not fluent. It was not untroubled journey when I had to use the English language which is very different from the Italian one. I had to adapt concepts and my own thinking process to the English syntax and therefore the text might sometimes looks unnecessarily intricate or boringly naive. We can agree that languages shape our thoughts. Thereby the reader might here and there observe my little inner battle trying to fit Italian ideas into English clothes. Said this, I start the exposure of the research giving to the reader a general picture of me, in the hope that this could help her or him to better get into the process I went through.

The journey that has guided me into the develop of this research has roots into my personal history as a musician and into my scholar times. I started the study of classical guitar at the age of eight. Nobody has forced me to approach music, *au contraire*, I was discommended to fall into the spider net of music. Many of use, I believe, have experienced the magnetic strength of music, which, especially regarding kids, attracts humans like bees on honey. My parents have experienced that and honestly they were not the most encouraging when I said them: I want a guitar. However, even cloud has a silver lining: when I decided to get a guitar at any costs, my parents agreed to bring me into a music school. There I started to face music in all its disciplines and I was introduced to the classical guitar.

At that time I listened to Pink Floyd, Led Zeppelin and John Lee Hocker but simultaneously I played John Downland and I got crazy with solfeggio. Despite the calvary of solfeggio at the age of eight, I decided to go on with the study of music. I wanted to discover what was that magic about, how did it work, which was the secret receipt for music spell. So did I and later on I started my academic studies withen the classical department of "Conservatorio G.Verdi" of Torino (Italy), where I graduated as a classical master level performer with *maximum cum laude.* In the academy I went deeper and deeper into the study of classical literature as long as music history and classical harmony. I kept on investigating on the mystery of music: on one hand examining how did the Masters fix their ideas on scores; on the other hand listening to the most various rock and jazz music and studying scales, chords, harmonies.

My hunger of knowledge drove me to study more and more harmony tractates, it drove

me to approach composition and finally to discover improvisation. Parallelly to the classical studies I started to approach jazz music and improvisation when I was sixteen. I started to play in jazz combos as well as a classical guitarist and as an electric guitarist; I learnt the interplay process within improvisation and the be-bop language as well. After the conclusion of my classical studies in the academy, I then decided to undertake the jazz academy path. I felt that improvisation, interplay and the idea of *personal language* were unfailing ingredients to music secret receipt. I graduated with *maximum* in jazz guitar as well.

Meanwhile this period of studies I of course grew up. There was where I started to develop my personal language as a crossover musician, now playing in different constellations and approaching different systems in music. There I had the opportunity to play lots of different contemporary music and various styles as well as the arabic one which has gave me so much inspiration. Lately I have toured with different bands and musicians coming from jazz and also classical backgrounds. I have done many collaborations with contemporary dance companies as well as recordings as a leader and co-leader. My parents were right: music net captured me that day and never let me leave *her* (music is a female name in Italian, I want her to be a *she* even with English semblance).

When I moved to Sweden in 2015, I was already a professional affirmed musician still trapped into the investigation of music. In Gothenburg I had the time to focus on a shade of that spell, on an ingredient of the special receipt: how could have I describe music? How was it constituted according to my studies and my experience? Last but not least: how could have I put some new spices into it? How to find *my secret ingredient?* I started then to recall the steps I have walked in my studies, searching for *my questions*, trying to identify the issues that have engaged me strongly.

Among my studies, even if I had to play a lot of electric guitar, I always kept the classical guitar as my main instrument and I was always very interested into composition and contemporary music. Since I have started to study music I was always supported into composing, first just some simple kids melodies then more complex compositions. As said before, my musical story starts with the study of classical guitar and its own repertoire, from ancient lute music transcriptions to music from the second half of the XX century. Studying it I had the opportunity to face a lot of different composed music, therefore I had the chance to observe different ways to organize notes and create music.

I was, since the very beginning, exposed to a lot of different systems to organize tones. When I was working on the contemporary music for guitar (which I enjoyed a lot and which is still inspiring to me into different contexts) I had the opportunity to bump into a lot of researches on techniques made in order to increase timbre and expression possibilities of the guitar (commonly called extended techniques). Together with these extend techniques I came into contact with a lot of different researches on harmony systems others than tonality. Since I was trying to compose on the instrument first, I faced lots of the limits and possibilities that my instrument implies. My experience of music has always been mainly practical. I have always tried out ideas and compositions in concerts and different performing context.

Some steps further into my memory lane I ran into jazz. Thanks to the study of jazz

harmony and language I came into close contact with the concepts of music grammar and syntax both theoretically and practically.

I want to stress here already that **Music grammar and syntax are core concepts in this research.** I start all my reflections from them.

In jazz it is very important to be able to build your own personal way of *speaking* (your language) and to be able to communicate with the other fellow musicians (which means to be understood, so to create phrases with a definite comprehensible syntax).
In this work I will use this comparison between music and verbal language; I will start from this similarity to define the *axioms which rule my twenty four notes based system.*

Studying improvisation I started to feel the need to increase my tools on the instrument and organize them into a full-working language. So I reflected upon how to build structures which allows me to be understood and personal at the same time.
How to create your personal language and still being understood by others? How to create a *really personal language?*

In this research and in that period of my studies I really focused on this similarities I found between language in jazz improvisation and verbal language. This path of *researching a personal voice* into music has led me to the search for new ways to organize music and new ways to play it on my instrument.

Coming really close to present days, I want to recall a trip I made to Istanbul. It was enchanting. I still remember the sound of that huge city, the colors of Bosphorus when the late sun is slowly lying into the turquoise water. There I had the chance to listen to some new sounds I had never listened before: the *quarter tones.* It was the first time I met arabic music into a close distance. I bought an Oud and a Saz and quarter tones sonorities never left my thoughts and my ears.

Some years later, in Gothenburg, I started to listen to a lot of arabic traditional music. After the listening of persian and iraqui traditional music I got enchanted again by the usage of quarter tones. I immediately recognized myself and my guitar into that sound. I had then the idea to try to get the flavour of that world and enriched sound, bring it to my music and my classical guitar, *but in an original and independent approach.* I started then to think about playing quarter tones on the guitar and generate a system to compose and improvise with quarter tones.

Since I remained linked to the classical guitar and to the acoustic sound I found in quarter tones the possibility to expand the timbers and melodic/harmonic items in my music. I immediately saw the quarter tones as a natural and faithful development of the resources of my acoustic sound.

In jazz music (but I believe generally in every kind of music, especially the ones which involve improvisation) it is really important to define a context that can be used and shared by musicians which are playing together. Therefore I wanted to systemize harmonies and

scales with quarter tones. I wanted to generate a context where it would have been possible to use quarter tones scales and chords into tonal derived music.

This process has brought me to the develop of few ideas before the moment I decided to go deep into the creation of a twenty-four sounds based music system. First I started tuning my instrument with a particular system which will be explained later, then I created a system which generates scales and harmonies that use these new sounds both in new compositions, both in already existing tonal music.

Came up until the very present days, I have to jump back when I was a kid again. Before to go serious into the hard studies of music, I was really interested into Math, Geometry and patterns. Thanks to my father who is a doctor in information science, I had the opportunity to get in contact with the taste of Logic, Geometry and Math. I always kept with me some of that love for rational formalizations and geometry properties. I have always combined my passion for geometrical drawings and music patterns. This research bring up some of that aspect of me as well and, as the reader will observe later in the text, many of those issues have been relevant to the definition of this work.

Before to go further with the exposure of the research, some thanks.

I want to thanks Valter Degani for his contribution on the mathematical formalization and the writing process with LaTeX and Lilypond usage.
A very special thanks to Diana Torto and Michele Rabbia who shared this music with me and to Anders Jormin who has always sustained me in this work.

Chapter 2

Introduction

If the previous chapter was meant to be a general and personal overlook on the person who has undertaken this work, this chapter aims to be more schematic and addressed to the treatise of specific issues which had to be considered *ante* the actual research.

In this chapter there are presented the cardinal points of the research, there are discussed the axioms which have been considered in order to define the edge and the specific ambit of the work.

As the reader will soon observe, this research is enclosure into a strictly technical ambit, however it claims discussions on aesthetics and on the definition of music itself as well. Thereby we can not refuse to face some debates which do not aim to be the goals of the research but which irremediably emerge, demanding us to array from one or the other side of the discussion.

However, even if this chapter is meant to contain the reflections aside the research, they will anyway emerge along the whole text, popping up as reflections parenthesis among the exposure of the actual work; as glimpses from the window of a running train.

Furthermore, concluding this premise to the chapter, here there are exposed the *why*, the *what*, the *who* and the how of the work. I want the reader to find here a little *résumé* of the text that follows. This section might then also be a guide for the reader: it is here exposed the structure of the whole writing.

2.1 Structure of research

Here it follows a list of core concepts and processes that can be considered as a summary of the research. These concepts are both goals to be achieved (or differently said, they are prearranged goals of the research) and both initiated issues later on partially and often completely achieved.

I start with some definition first.

- This research consists in the creation of a method to organize a system which uses

twenty-four notes into a tonal functions context.

- The aim of this work is mainly practical, I wanted to compose and play music with quarter tones carrying a different approach from the tradition of Maqām and classical European music researches on microtonality.

- This work aims to create a base to construct a new method both to compose and improvise, especially into a jazz-derived improvised music context. The material here presented is therefore useful in traditional tonal choruses and it can be studied and used into several music genres.

In order to approach those issues I adopted some processes in which I go through now.

Firstly I have defined a method which I have used to get results that have guided me towards the definition of a system to organize twenty-four notes music.

This method has a deep connection with geometry and it starts with the consideration of music and its procedures through a mathematical formalization.

Later I have used the method to compose some pieces and I was involved into a recording session where I tested out all the composition and the new improvisation features.

The method is articulated into three steps.

- In the first step I go through the basic concepts which rule the running of the method. There I define the ambit of the system and its basic properties. As explained further in chapter 3 this system deals with the concept of tonality expanding its domain through quarter tones. Here I propose many reflections upon music syntax and therefore semiotics.

- In the second step I adopt a geometric concept towards a *generative and quantitative process* which generates scales. There I propose and collect a lot of basic material.
  In this step it is created coherent material that still has to be organized and interpreted.

- In the third step I define, and later on apply, principles to make decisions into the material coming out from the generative process. There I adopt a *qualitative process*: I decide what is meaningful to my music system.

Finally I compose music trying out the material.
I use the method proposed to compose music I personally relied on, fulfilling it with some personal aesthetic and artistic choices.

After this *summa* of the research I want here to spend some words to express the personal needs that led me to take the decision to approach this subject: *why* did I choose it and which are my associated *goals* in it.

I decided to meet this subject because it came as a natural development of my personal music journey.

As the reader has probably observed in the previous chapter, quarter tones has been being part of my artistic *bagage*; therefore this research is born as a need to systemize those sounds into a more useful and coherent way. Furthermore my artistic choices have always involved tonal language as long as definite *musical notes* and narrative development of music. It was necessary to me then to conceive a system possible to generate tonal-derived music with quarter tones; a system that could have allowed me to identify new harmonic and melodic tools to approach functional harmony.

I consider therefore this research as a tool to develop my language and my music.
Finally the *why* of this research is enclosure into my melted identity of classical/contemporary/jazz musician which struggles to find a personal way to express his own artistic voice into the contemporary improvised music.

The *goal* in this research is finally to create a base to the developing of a music language that uses quarter tones both to compose and to improvise which could be used in every kind of music. Generally, the goal in this research is to create a system which allows musicians to compose and improvise with quarter tones.

Before to face the room of reference of the work, I want to help the lazy reader, as well as the impatient one, proposing a guide to navigate among the whole work. The ones who feels like it, feel free to jump across the writing and look directly in the section that suits them the best.

The core of this work is enclosed in chapters 4, 5, and 6.
There I explain my methodology (chapter 4), I use it to get results (chapter 5) and finally I use those results to compose pieces which I here analyze (chapter 6).

Together with these chapters, the audio files are fundamental to this work since they represent an application of this method in music, which is the practical essence of this research.

In chapter 3 I face some semantic issues which I had to face into the definition of the base axioms of the method.

In chapter 7 there is an explanation of some practical issues I faced into the work.

In the last chapter I analyze the results and pose further questions and lines of development.

Finally in the addendum there is a brief history of researches about microtonality, along with all the practical features of the method.

This research contains graphics, scores and media files.

2.2 Room of reference

I now go through the issues that have emerged from the reflections upon the work, the issues that I have faced in the definition of the room of reference of this research. Before to start the discussion, I first list the axioms I have taken for grant and subsequently I argue my decisions.

This work proposes a criteria to make music which uses quarter tones. Here **I do not discuss the formation of notes, I take for grants twenty-four definite notes and I approach the question of how to organize them.**

The method used to generate scales and harmonies is a geometry based method which deals with *definite pitched notes into a finite set of twenty-four objects (modular operation).* Here I do not propose music which does not use notes. *Here I propose tonal derived music with quarter tones.*

Both method and music proposed belong to tonal derived music organization and they rely to the similarity between music syntax and verbal syntax.

This research involves different steps of music process:

- The analysis of tonal derived music systems as a modular system, explanation of scales and tonal functions as additive functions. Further the comparison between tonal music structure and language structure and therefor the analysis of music as language divided into level of structures and ruled by syntax boundaries.
- The construction of a generative geometric method to determine scales and the usage of a qualitative process to build tonal scales into a twenty-four notes context.
- The composition of music which makes use of the material gained from the method and the proposal of a personal artistic aesthetic of quarter tone music into a post-jazz improvised music context.

I will go now one step by one discussing the theoretical and musical context of this work.

Firstly the work starts considering music as language and music syntax as comparable to verbal syntax. According to Leonard Bernstein [1], I consider music as language and I compare tonal music grammar to verbal syntax. [2] As said before I here approach tonal derived music, I do not consider any other kind of sounds organization in music. Furthermore, I deal with twenty-four notes (which I arbitrary define as quarter tones) and here I do not discuss temperament in music. Influenced by jazz, I try to construct a scales and chords syllabus which allows to use quarter tones for composing and improvising where there are *tonal centres and cadences.* I am interested in creating *key points* which allow to interpret the other notes in relation to them, as it is done in tonal music. Said this I start an analysis of tonal music as a modular system where tonal scales can be described as additive functions and preferred

[1] Leonard Bernstein. *The Unanswered Question: Six Talks at Harvard.* Cambridge: Harvard University Press, 1976.

[2] There will be dedicated a section to this issue in chapter 5.2

order of movement is defined by rules of grammar, gluing together music, semiotics and Math.

There is a big discussion upon the issue of music as language and problems connected to the lack of semantic meaning in music as in verbal language. I mainly avoid to take to part to this discussion since I do not want to elaborate a system to *explain music*; I rather want to analyze only a specific way to create music: the tonality. Therefor **I do not deepen the discussion regarding the issue upon the question of: "if music can carry a semantic meaning"**, but I analyze tonal system rules as grammar rules. By the way, since I finally compare music to language, I place myself into the ideas of Ugo Volli [3] whose proposes an extended concept of semiotic which allows music (together with all others artistic languages which does not carry a precis semantic meaning) to carry a specific semiotic content. Even if we may argue upon the interpretation of music messages (does music express some kind of content which can mean one and one only meaning? Can everybody agree upon what *that particular* piece of music *mean or express*?) we can agree that *music express a content.*

Regarding these assertions, I want now to initiate a brief reflection upon the concept of *signe* according to the reflections of Roland Barthes [4] (and, as reported in Barthes writings, Saussaure).

Barthes affirms that the concept of sign enclosures the two concepts of *signifié* and *signifiant*; he affirms that whenever we talk about signs we talk about a concept constituted by the union of form and content. [5] According to him each sign requires a form to be represented and it has to carry a meaning to be communicated. Furthermore he proposes a dialectic relation between language and *parole* (recalling again Saussure); he argues that it could never exist any language without its current conjugation; it could not exist form without the application of it and of course viceversa.

I start from this two reflections in order to propose a variant of its concept of sign in the artistic ambit which is relevant for this research.

Even if I can recognize a division between *signifié* and *signifiant*, according to Barthes, I can not accept that they could ever exist as an isolated entity, therefore whenever it exists a content it exists a form and whenever it exists a form it compulsory exists a content (defining

[3] Ugo Volli. *Manuale di semiotica.* Bari: Laterza, 2003, pp. 264-267.

[4] Roland Barthes. *Elementi di semiologia.* Torino: Edizioni Einaudi, 1966.

[5] This classification recalls the one adopted by Aristotle which, as stated by Umberto Eco in *La struttura assente*(Umberto Eco. *La struttura assente. La ricerca semiotica ed il metodo strutturale.* Milano: La nave di Teseo, 2016), can be considered former father of the structuralism. Aristotle individuates three layers of schemas within an *ontological* (and probably also *epistemological*) consideration of reality. He talks upon the *morphé* ($\mu o \rho \phi \acute{\eta}$) which can be defined as the physical shape of the thing; the eidos ($\varepsilon \tilde{\imath} \delta o \varsigma$) which is the idea, the paradigm of the things or in other terms its class quality; the ousia ($o \grave{\upsilon} \sigma \acute{\iota} \alpha$) which is the union of morphé and eidos, the thing itself.

Differently by Plato, he does not accord to the eidos the capacity of existing by entity itself; eidos and morphé always exist together.

We might therefore compare this classification to the Barthes' one: *signifié* ($\varepsilon \tilde{\imath} \delta o \varsigma$) and *signifiant* ($\mu o \rho \phi \acute{\eta}$) are always together in the sign.

a dialectic of the *signification* between the $\varepsilon\iota\delta o\varsigma$ and its $\mu o\rho\phi\eta$).

This assertion seems to claim the division between *methodological structuralism* and phenomenology proposed by Eco [6]. The structure is a grid we juxtapose to reality in order to describe it and understand it; thereby it is never possible to separate morphé and eidos, otherwise we would fall into the trap of the postulation of an ontological structure which shapes reality. It is rather the other way around: we can verify the schema which emerges from reality and which can thus describe it.

Even if I array myself to the methodological structuralism, I want here to propose a different reflection on the dialectic between *signifié* and *signifiant*.

The previous assertion seems to let me be in accordance with Barthes, but let me now argue this assertion. My reflection starts from a question: has not the *signifiant* a *signifié* itself? Claiming the ideas of Kandinsky [7], I consider as meaningful the signs themselves, I assume that a particular *signifiant* has a meaning itself. The decision to adopt one *signifiant* or another one signify itself something. Thus, the form it has decided to transmit a content can be considered a content itself as well.

Furthermore every act which consist into a voluntary choice of a sign (its symbolic, representative part, its *signifiant*) involves meaning itself. We deduce from this that it can not exists any *signifiant* without its *signifié* since there is *signifiant* whenever there a voluntary choice to communicate a *signifié* through a specific *signifiant*.

However I argue here that we can identify a double level of signification: the *signe* makes use of the *signifiant* which in turn signify as a voluntary choice of a symbol rather than another one.

We can deduce thus that every language is *polysemic* even into its structure.

Surely the reader can argue that the arbitrary decision to assign to the *signifié* of "home" the symbols (*signifiant*): "home" ; "dom" ; "dtfgr48", constitute itself of a double meaning choice.

Therefore in this sense (as Barthes proposes), the *signifié* belongs to the *signe*, it does not place itself outside of it.

I quote from him: [8]

semantic has to belong to structural linguistic, since the ***signifié*** *belong to signs , when according to the American mechanists the* signifié *are substances which have to be ejected from linguistic and moved towards psychology.*

[6] Umberto Eco. *La struttura assente.*
[7] Wassily Kandinsky. *Punto, linea, superficie.* Milano: Biblioteca Adelphi, 1968.
[8] Personally translated from: Roland Barthes. *Elementi di semiologia*, pp. 29

This reflection will be useful later on to conduce further reflections upon the semiotic meaning of arts.

Defined this coexistence of *signifié* and *signifiant*, we will be thus able to hypothesize a signification of the signifiant and therefore a semantic meaning of music (in the follows, this principle will be represented as Sf↔Sn, where Sf refers to *signifiant* and Sn refers to *signifié*).

I now go further with this reflection proposing it in a Gnosiological and Epistemological ambit. These reflections drive us to affirm that there can not exist any concept if not represented. It can not exist a content without a code to express it. Again, it can not exist *signifié* without *signifiant*.

Consequently everything depends on the language. The reality can be understood only through its representation, therefore we can not refuse to rely the concepts to a form that allow us to conceive them and later on to communicate them. Once again: it does not exists the idea itself separated from its representation [9].

I quote again Barthes: [10]

[...] there is nothing real which is not intelligible [...] and again *the signification is not the real thing, neither its psychic representation, but the way we say it. It is neither act of knowing the reality, it can only be defined through the process of signification, almost in a tautological way: it is that something that whom utilizes the sign implies with it.*

After this premise, having thus defined the relation Sf↔Sn [11]; we can therefore affirm that whenever one exists, the other one will always exists as well.

Thus, even if we can identify only the *signifiant* (as it seems in paintings and music), it has necessarily to exist also a *signifié*; definitely not through the process of denotation but surely possible to be postulated according the principle (just proposed) of dialectic interdependence between *signifié* and *signifiant*.

Newly: whatever human voluntary act (from the proxemics to music, to verbal language) signify within a cultural context, but to signify/*signifié*/signification they all imply interpretation or better de-codification i.e. communication.

This is an important statement: whenever there is signification, there is communication; it can not exist signification without communication.

The choice of a particular cloth communicates or better signifies since someone else than me is able to interpret the act, notice the act. Even if the act does not imply a direct communication (I choose the cloth for my sake, in order to go out), it actually imply a receiving

[9] I do not mean that this representation has to be verbal. Simply in order to conceptualise an idea we have to compulsory elaborate a representation of it and therefore to define a form, a *signifiant* to it.

[10] Personally translated from: Roland Barthes. *Elementi di semiologia*, pp. 32-33

[11] The symbol ↔ is here intended as interdependency between the two elements involved.

(to dress signify because they look at me). [12]

Concluding the reasoning, we can assume that if reality exists itself even if we do not perceive it, the act of perceiving and understanding it is strictly related to the act of communication.

Here I want to jump back again to Eco's reflections upon structuralism. Even if Aristotle is postulating the ontology of the *ousia* and therefore the ontology of the eidos; it is discussed if the eidos exists in the things (reality) or if it exists only in our representations.

Since we can not elaborate, perceive reality without a representation, I believe that the ontological reflection just proposed is suddenly overtaken by an epistemological and gnosiological problem.

We will never be able to approach reality without our logos ($\lambda\acute{o}\gamma o\zeta$) and sensibility; therefore it is not relevant to me to understand the characteristics we give to schemas, the codes of *our* eidos and if they are common out of culture communities.

Since we elaborate a concept of reality, we shape it through the language, we define a signification. Thus the act of thinking and generally approach reality is social in its structure.

I conclude this part of reflection saying that this might drive us towards the belief that human being is a social being and that the process of *knowing* involves *signification* and therefore it is always collective and towards the collectivity. Furthermore it involves communication, and therefore syntax.

After this reflection I go further quoting an important Italian musicologist. I quote Enrico Fubini [13]in *Estetica della musica*: *the* ***language*** *of sounds recalls the world of emotions and affections when it separates itself from the verbal one. Even if it is* ***polysemic****, even if it is sometimes ambiguous it is possible to perceive the content. It is a kind of language which comes before the language, a language which comes before the* ***denotative*** *quality of the word but which is full of recalls and resonances maybe thanks to an isomorphism of the sound languages to language of emotions and affections.*

And again from M.Imberty in *Suoni, Emozioni, Significati* [14]: *All speech about music is a metaphor [...] the music form remains a pure virtual language in which it is elaborated an* ***intention of meaning*** *which can not be transposed into verbal language phrases. From this the nearly infinite range of possible interpretations, their pertinence and at the same time their partial arbitrariness.*

Again I want to propose here a reflection upon languages functioning paying a special atten-

[12]In these assertions Anthropological studies meet semiotics; since every act recognized by a community can be considered a signification act and both its code and structure (semiotics); and both its relation to the cultural context (anthropology) can be studied from these disciplines. Especially Levi-Strauss has undertaken a lot of studies later on re-considered by semiotics.

[13]Enrico Fubini. *Estetica della musica.* Bologna: Il Mulino, 1993, pp.32-33.

[14]M. Imberty. *Suoni, Emozioni, Significati.* Bologna: Clueb, 1986, pp. 56-following.

tion to music and its information stream.

We can affirm that there is communication when there is a stream of information between two individuals. When there is a verbal communication for instance, the process always involves at least two people: one is streaming some contents voluntarily to the other one which adopts a code to get the information and later on elaborate an interpretation of the message. If when we think upon the verbal (or written of course) language it is clear what is edge between the reception of the messages and their interpretation; in music there is not such a clear separation between those. We can all argue that music *employs an effect* (as stated by Ugo Volli) on the receiver, however we can not easily place the moment when this operation takes part. Is the effect set in the message itself (is it its content) or is it set in the personal interpretation of the receiver? Did the composer mean to generate a particular *affection* on the listener, did it the interpreter or is the affection only a result of the fruition of the music?

We might assume that music could therefore be defined as an *uncontrolled communication* (or "*kind of language which come before the language*" as stated by Fubini); a stream of information which determines an always different content depending from the occasion.

This might let us think that music language has no *signifié* but only *signifiant* when the *signifié* belongs to the act of interpretation.

Also Barthes defines music as possibly *semio-genic* and not *semantic* since it should belong to the un-conscious area of the *pre-sense*. However I claim my critique on Barthes previously faced and I recall back here the enunciated principle of Sf↔Sn.

It follows that it has to exist a *signifié* belonging to the just discussed *signifiant*.

Let us suppose that music does not carry a precis *signifié*; but how, if the music would not carry any precise content, could the listener decode the message? How could the composer as well as the interpreter drive music to some directions?

I believe that (as I will quite widely argue further in the text), music communication is not a *random communication*. I believe that there is not only a syntactic structure of its code, but I do believe that music can carry precise contents which, when decoded by the receiver [15] allow a multiple possibility of interpretation (much wider than in the verbal language).

This content is the form itself, it is the structure of the music itself, its harmony, its phraseology, its rhythm features and so on.

This assertion will be argued plural times in the text, for the moment I approach it just from this corner.

[15] Of course the decoding process does not involve a personal interpretation of the message, it is incontrovertibly defined by the essence of the code itself: it is enclosure in the language

Again, this particular essence of music (the less representative language among others) led to the certainty that music does not carry any semantic meaning, that music is somehow totally depending from the listener. *Au contraire*, I array to another position into this debate. I think that music can be driven to different contents and I propose a different balance between decoding and interpretation in music.

The reader might here observe that in poetry already the edge between decoding and interpretation becomes more labile. Whenever the *sound* acquires importance, the semantic property of languages seems to slowly become evanescent. Nevertheless we can not deny a code to rhythm in poetry, we can not consider only the verbal content of it. Doing this we would put poetry together with all other kinds of written text and therefore we would not consider poetry as a language itself. But newly, rhythm in poetry has a meaning which is expressed by its structure. The way we decide to shape the rhythm (and later on also the prosody) into our poem, affects the *meaning* of it. Subsequently it generates different effects on the readers which will interpret it differently. Nevertheless the rhythm remain the same, maintaining its properties and its functionality.

The same we can of course say for music.

After this reflection, might the reader let me go slightly further with the consideration of music as syntactic structured language.

I premise a clarification: As said previously, in this research I approach tonal-derived music. Nevertheless someone can argue that music, if considered as general phenomena defined by a voluntary and arbitrary organization of sounds in time, can not carry precise semantic meaning. However, I want to narrow down the reflection; let us focus now on tonal music: is it possible to identify mistakes? Can we say that, defined a tonal context, there is some *wrong notes*? I am quite sure that the reader might agree upon the fact that, in tonal music, there are *correct* and *uncorrect* notes, depending on the context.

Let me propose an example.

What if as harmonic support of the following melodic line:

C - D - / E - F ;

I place the chords:

G7 and C ?

The professional musician as well as the common audience will definitively perceive an incoherent message, a *cluster* of information between melody and harmony. While the melody proposes a movement from the Tonic area to the Dominant or Sub-Dominant area; the harmony is proceeding in the opposite direction. The receiver of the message has not to know the rules of tonal harmony to perceive this unhappy juxtaposition of contents.

These assertions might be argued by this affirmation: We always see what we can recognize, we see what we know, what we are able to see.

Possibly a listener from a Bushmen will not recognize the cluster of information just discussed; it is hard to affirm that there is some universal key of reading music. It is hard to agree upon the fact the there are some *gestalt* that every human being uses in order to approach music communication. It is more reasonable to say that music belongs to culture and therefore it is totally depending from the cultural and anthropological context to which it belongs. However it is part of occidental culture tradition to search for some *topos*, some archetypes on which relate our thinking. The history of intellect is sprinkled of issues in which intellectuals always arrayed from the relativism party or the absolutism party. Humans always searched for some paradigms to fit the reality into them; some has believed in the existence of some of them, others have denied everything out of the analysis of facts and the sensitive experience.

By the way, the semiotics and the linguistics study the structure which can be deduced by communication and trough comparation they try to identify some principles. Thus, even if communication codes, *signifiant* and meaning of determined *signifié*, belong to a community, to a determined culture; we can argue that the essence of languages exists above those. We can argue that the essence of languages is not dependent by its conjugation.

Nevertheless, the theory of the *universal grammar* of Noam Chomsky, concedes to languages some *gestalt*. Chomsky is talking upon verbal language, however, in my opinion, his thoughts might be transferred to music as well. Therefore I answer to the critique in a double way.

Firstly I here state that this research, as well as the study of semantics and linguistics, investigates the structure of communication itself, as an act. Here we are not undertaking a reflection in the cognitive side of the issue: we do not want to affirm that music employs the same effects to every listener and furthermore that it means always the same. We just argue that there is possibly some *invariant* structure into music language; a structure which belongs to the code of its communication.

Moreover, on one hand I sincerely believe that every time it is set a context of meaning in which there are given some arbitrary parameters on which music is built, every listener, even if she or he does not know them from before, can recognize them and therefore react to them. [16]

On the other hand I answer the critique setting in a better way the context of the research. Let us suppose that music is totally relative and that it depends totally from the cultural context: my research will therefore belong to European music, it will be useful to European listener and musicians and it will refer to the European music tradition.

[16]Since this dissertation is not central to this research, I have not deepen into it, I have just reflected on the issue and for the moment have given myself this answer that might be confuted later on.

After this little parenthesis I move on with the reasoning just interrupted, concluding it.

The previous music example and the following observations might drive our convincements towards the assignation of some *semantic* contents to musical messages in tonal music.

Let me now introduce another step towards the consideration of tonal music as a semantic language. Tonal music seems to be able to generate a context of interpretation of notes which let anybody possible to interpret music developing and *predict* it. Tonal system orders notes according to partial order boundaries, it suggests coherent developing of the lines according to the rules of harmony. This means that it allows the listener to make hypothesis on the possible direction of harmony and melody, giving expectations.

Let me propose another example.

Let us consider these two harmonic progressions:

$$\left\{ \begin{array}{ccc} D-7; & G7; & C \\ II; & V; & I; \end{array} \right\}$$

and

$$\left\{ \begin{array}{cccccc} D-7; & G7; & Amin; & Abdim7; & F/A; & G/B; \\ II; & V; & VI; & VIbdim; & IV/VI; & V/VII; \end{array} \begin{array}{c} Cmin; \\ Imin; \end{array} \right\}$$

The first one follows the most simple path according the expectations of the listener; her or him will perceive an untroubled flow, a simple and clear phrase.

On the other hand, in the second progression there are some unexpected *cadences* which (even if they are correct according to the harmony rules) surprise the listener whom will follow the direction of the speech comparing it (movement by movement) to her or him expectations.

Can we not consider this a semantic communication?

Since there is both the possibility to write a *wrong* (meaningless, incoherent) message, both to consider different levels of connections (different meanings); can we assume that tonal music carries semantic contents?

If it is true that music can not communicate a precise content regarding issues out of music itself; we can argue that tonal music transmits contents that are not depending from the interpretation of the listener when they belong to music itself and its development.

We can therefore reflect upon concepts such as tonal grammar and syntax; we can analyze the developing of their usage in music history and try to understand *why* and *how* are they generated by tonality. We can finally try to build systems which are able to carry these tonal functions.

Furthermore, even this reflection might contribute to explain why tonal music has never been overtaken by other systems to organize tones such as dodecaphony.

Regarding this property of tonal language I want to propose here briefly a reflection from Chomsky.

In *Linguaggio e problemi della conoscenza* [17],

Chomsky postulates the existence of some rules belonging languages which are proper of the human being mind processes. He hypothesizes an alien scientist which is trying to learn human language deducing from empirical experimenters, so he shows some peculiar properties of languages which are structural related (and hierarchically organized) differently from the most linear possibility of language structure.

He therefore affirms that there are some rules of language which are innate into human beings and that these rules are not belonging to logic/rationality, they just belong the some inner structure of all human languages.

I quote him: [18]

We can easily construct languages which make use of linear rules, simpler when it comes the computational process. [] Those languages would work perfectly good in order to communicate messages, but those are not ***human languages.*** *Kids would occur in much more difficulties into learning those languages, and adults would as well find much more hard to use those instead of the more complex human ones since they should adopt conscious computational rules instead of relying on instinct.*

This assertions might be driven to tonal music as well, it seems even though it is possible to determine further notes organizational systems, that tonal music carries a special strength, such as human languages for Chomsky.

By the way, I will not debate upon the issue regarding why tonal music carries these special properties (if is there any physical, natural base); I anyway take for grant these special qualities and these semantic properties.

Before to move on to the discussion of the second point of this research I want to propose a brief reflection I will not treat in the research [19] regarding the *purpose*, the *aim* of music and generally artistic communication.

Preliminary to the treatise of this subject I want to introduce the concept of *aesthetical*

[17] Noam Chomsky. *Linguaggio e problemi della conoscenza*, Milano: Il Mulino 1991.

[18] personally translated from *Linguaggio e problemi della conoscenza*, p.43

[19] In this research it is relevant to analyze music syntactics and its code. It is relevant to approach the issue regarding its functioning and tangentially what does music could mean and how. It will not be deepen, except in this short parenthesis, its *purpose*, its *function*.

idioletto .

According to Umberto Eco [20], the *idioletto* can be defined as the set of norms agreed by a determine community in the interpretation of an artistic code.

For example if we consider the paintings in the Europe of the XIX century, we might argue that there were some socially accepted codes of making a picture. Whenever there would have been a deviation from them, the user of the art piece would have react to them.

We can thus define the *idioletto* as a convention accepted by a community regarding the code of art and consequently the meaning of art. Now, since Eco defines (according to Jakobson) the aesthetical communication as deeply ambiguous, there is this way determined a paradox between the norm of *idioletto* and the ambiguity of art. Eco approaches the problem proposing two points: [21]

- the artistic communication can never be reduced to a structural analysis of it.
- the artistic communication has to carry a *structure*, otherwise it would generate random answers.

He therefore approaches an issue which has just been debated previously in the text regarding the edge of interpretation and de-coding of artistic languages, and therefore the problem of the *purpose* of art. Quoting Hjelmslev (1928: 240) Eco writes: *" The extra-grammatical or affective elements can actually be ruled by grammatical rules, and maybe by some particular grammatical rules which has not been discovered yet.*

He is thus claiming the existence of a syntactic structure even in artistic messages, but let us go back to the issue regarding the *purpose*. Eco later on proposes the opinions of the Russian formalism, quoting the term of *effect of disorientation*. For Sklovskij (1925), the purpose of artistic communication is to disorient the receiver in order to make him reflect upon the language itself and *relata* sign/reference.

Again from Eco:

[...] the purpose of the ambiguous and self-reflective message can be considered as ***way to the knowledge****.*

Therefore we might resume the reasoning in this way. Art carries ambiguous messages which drive back to the code itself. This is a peculiar characteristic of art languages, it is not accidental. The *purpose* of art is therefore to be ambiguous and consequently stimulate the receiver to interpret the message using connotation (semiological terms will be explained in section 3.1; please have a look there if you are not familiar with them). Thus they propose new connotation between references, they expand the paradigm of signification trough ambiguous syntagma. Therefore art employs an evolution of the rules of the language, and brings to the receiver to pose the question: *why does this sign denote/connote this reference?*

[20] Umberto Eco. *La struttura assente*; pp.114-115
[21] Umberto Eco. *La struttura assente*, pp. 117

Why does it exist this relata?

Consequently we can affirm that art is strictly depending by culture, since the *idioletto* is a cultural product and we can finally observe that art is ***motion of questionings*** and thus going forward process into the language.

Art poses us in front the questions of the language itself, it makes us question regarding its nature and its functioning going again back to the root of it. We face the code and its *mise en discussion* and therefore we propose new possible contents and new possible connotative meanings.

Finally art could be considered as social/antropological item to discuss language and signification; it could be considered item to propose new cultural contents into the process of signification of reality and therefore it stimulate the gnosiological process and pushes forward our conception of reality.

As said before these issues are not object of this research, however I think they are relevant approaching the discussion of music as language (which finally is a core concept in this research).

I now go on explaining the context of my second point, the creation of the method.

Starting from the proposal of tonality into a mathematical formalization and the description of tonal music as a modular operations context, I adopt a geometric tool to organize notes and generate tonal functions. More precisely I firstly set all the notes chromatically into a circle, then I make use of geometric figures to determine a selection of notes. This double link to math (regarding the ontological exegesis of the concept of scale) and geometry (regarding the generative process of scale construction) can be set in the tradition of Nicolas Slominsky and Masaya Yamaguchi. [22]

In the generative process I especially take moves from Slominsky in *Thesaurus of Scales And Melodic patterns.* [23] He divides the twelve tones into equal portions and he generates scales starting from this selection of notes. He uses only perfect divisors and he of course works into a twelve tones context. I start from his concept and I develop it into a twenty-four notes context and by using different symmetry criteria (not only perfect divisors).

I set my research *à côté de* Slominsky's framework even though I propose here an explanation of his work through the usage of new processes that will be explained in details in chapter 4 (I do not discuss them here since I consider them as actual part of the method, as actual part of this research).

These processes have provided me a lot of material that had to be chosen and organized into a useful frame.

[22] Masaya Yamaguchi. *The Complete Thesaurus of Musical Scales.* Masaya Music Services, 2006.
[23] Nicolas Slominsky. *Thesaurus of scales and melodic patterns.* Charles Scribner's sons, 1947.

I decided to adopt a geometrical principle to generate scales because I believe that it is efficient to bring in music inspirations from other disciplines. Many artists have done this, such as Kandisky and Mondrian, and regarding this I want to quote Boulez in *I neuroni magici*: [24]

The theoretical reflection seems to me particulary interesting when it is applied to something which is completely out of music, because it allows to discover solutions which could have never been founded if we would have remaineed closed into the set of our discipline.

Going on with the explanation, the method needed thus a partition into consequential sections: if I started from Slominsky into the definition of my generative process; I take moves from George Russel and partially Shönberg in its second part; the qualitative process.

Therefore, later on I adopted a evaluating process to organize and generate scales using the concepts of *tonal area* and *diatonicism*.

I quote Russel in *Lydian Chromatic Concept of Tonal Organization*:
The tension between the C Lydian and the C Ionian is responsible for the state of duality evidenced by all three major scales structures. [...] They are non-final harmonic structures permanently fixed in a ***horizontal linear time state*** *as non-final chords forever evidencing the tendency to* ***resolve to a final sounding cadence center or goal.*** [25]

And again:
The major scale represents the horizontal, musical active force ***forever in the state of resolving*** *to its I major or VI minor tonic station goal (cadence center) in the context of linear time, forever in the state of striving for unity with such goals. It defines the very nature of resolution as a state of fluctuation between tension and relaxation, non-final and final, occurring necessarily within a linear (horizontal) sequential time continuum. This is confirmed by the major scale primary harmonic progression. The ascendancy of the major scale to the position of Western music theory's seminal scale may be due, in no small part, to its manifestation of this most fundamental chord progression of the classic era. The major scale is truly a* ***diatonic scale*** *as [di] is the Latin prefix meaning [two]. It contains the C Ionian as the primary tonic on degree I, and the C Lydian on its* IV^{th} *degree.* [26]

I start from the Russel's analysis of diatonicism and I try to create quarter tone scales which can generate cadences between *Tonic* and *Subdominant*. Shönberg in *Harmonielehre* [27] explains the tonal system referring a lot to the concepts of *Tonal area* and Cadence, for him tonality exists when it is possible to generate movements between areas; therefor a tonal scale has to carry the diatonic property. I developed my scales according this principle: **I tried to create scales which carry cadences.**

[24] Freely translated from: Boulez, Manoury, Changeux. *I neuroni magici.* Carocci editori, Sfere , 2014. pp.158
[25] George Russell. *Lydian chromatic concept of tonal organitation.* Massachusetts: Concept publishing company, 2001, pp. 5,7.
[26] Ibid, p. 8.
[27] Arnold Schönberg. *Trattato di armonia.* Milano: Il Saggiatore, 2014.

These explanations seem to avoid any kind of inspiration from already existing music that makes use of quarter tones. Nevertheless there is a connection between my system and arabic Maqām. As we can argue, possibly every organization of a limited number of elements shares common results with other organizations.

Quarter tones tonal and modal music has been composed both in European tradition (see as a relevant example Alois Hába [28]) and of course in Arabic music. Again, my work **does not start from these traditions. As said before, it takes part from an abstract formal approach and it creates new composition items.** By the way, the method has generated also scales that can be found in tonal music and arabic Maqām scales. Maqām is a system based on *disjunct tetra-chords*; it uses quarter tones but it adopts a totally different approach to the creation of scales and it is not tonal, rather modal. My method is therefor different from the Maqām; not all the scales I found can be found in Maqām and viceversa. However Maqām scales are somehow related to tonal scales, therefor it is possible to find connections between scales in my system and the Maqām. The method has provided also a lot of scales already used by many different cultures in the world. Finally, a relation with the arabic classic tradition has been found in my work only at the end of the process, when I have used the result of the generative method to compose music; in same cases, in fact, I have been inspired by the rich tradition of the arabic music.

I discuss now the context of the third step: the composed music and my personal artistic aesthetic in using quarter tones.

When I started to approach this new method it was of course difficult to find some other music to relied on. The music I composed with the proposed method tries out different features of the system, it explores possibilities offered by it with different techniques. As the reader might has observed, I here focus only on pitches: harmony and melodies. Even though my music explores a lot also rhythm and timbers I am here interested into explaining the usage of quarter tones harmonically and into phrasing. Furthermore, there is a lot of improvisation and jazz derived phrasing as well as interplay. More briefly, I tried to transfer my language to this new world built up of new notes. Doing this, I got inspired by Persian and Iraqi music. From them I borrowed rhythm features and phrasing, as well as Maqām scales together with the scales generated by the system. Again, I widely use improvisation as in the European jazz tradition and I apply here original composition items which **I have constructed for my music.** In some pieces there will be traces of arabic tradition blended with my personal voice. In others compositions I try out a new experimental post-tonal chord progression with quarter tones. Generally I blend all the items together through improvisation. All the composition process will be deepen later in chapter 6.

Let me state more clearly an important consideration for this work. ***In this research I deal only with harmony and scales/melody.***

[28] Alois Hába. *Sonata for Quarter tone Piano op.62.* [online video], 1946-1947. Available at: `https://www.youtube.com/watch?v=s7vZURdhucM` [Accessed 15 Jan. 2017].

While in the composition process timbers and rhythms are central to my aesthetic, in this research I will not talk about them. These aspects are naturally blending in my compositions with the material coming up from this research. Naturally, I have tried out different types of compositions and I dealt with all the parameters of music, anyway fellow colleagues will shape the material to their own personal aesthetic, bringing in reflections on rhythm, timber and so on.

Even if it is definitely very interesting to catch the features I have constructed for my compositions, these are not the core of the work.

The compositions here proposed can be considered as one of the first attempt to compose music with the proposed method. I will hopefully compose new different music which will overtake these compositions here showed (I do not mean that they are not properly good ones, I just mean that they are only a first possibility of the method).

They are explained and analyzed in chapter 6, but please consider the fact that the core of this research is the method: through it, there will hopefully be composed new music; music which will carry different aesthetics and *gefühl*.

I now move on to the next chapter in which I face some other theoretical issues together with the first part of the construction of the method.

Chapter 3

Base of the method. Tonal scale as modular additive operation

We now enter into the actual explanation of the proposed method.

After some further reflections and a small semantics thesaurus, it is here exposed the concept of tonal scale as *tonal function*, introducing a mathematical construction that can describe it.

In this chapter there are thereby discussed the bases on which the methodology founds its roots.
They are here explained the basic axioms of the method and there is introduced the approach used later in chapter 4 and 5.

As written previously, before to approach the *semi- semiological* analysis of music as language, I propose to the reader a very brief discussion of some basic concepts of semiotics which will be useful to better understand the reflections upon our musicological analysis (some concepts has been already expressed in the previous chapter, nevertheless I think that it is useful to place this section here).

3.1 Semiotics: basic concepts.

As it is probably already clear to the reader, this research consist in an harmonic experimental *traité* which however takes moves from the consideration of music trough the glass of semiotics.

I have decided thus to clarify some basic concepts of the discipline, claiming mainly the manual from the great Italian semiotician Umberto Eco, *La struttura assente* [1].

Since I do not want to propose a manual of semiotic myself, I will structure this section with some couples of opposite terms, which are explaining some concepts I consider strictly relevant to my research.

[1] Umberto Eco. *La struttura assente. La ricerca semiotica e il metodo strutturale*

The first couple is:

- **Information signal/sense.**

  Eco starts its very clear and exhaustive explanation of semiotics explaining the process of communication in its essentiality. He firstly imagines the communication between two machines, two robots, which are instructed to receive and reproduce some basic signals in order to communicate one to each other. He proposes the example of a dyke in which it set a robot which has the work to measure the level of water and communicate it to another robot far away which has the work to produce some determined signals in order to communicate the information to a technician. The first necessary operation to be faced is of course to define some levels which can be recognized by the first robot and to which it will assign some pre-determined symbols. Of course the measurement of water is possible to generate infinite values, since water is *analogic*, so they have to be defined some steps. It has to be defined a digitalization of the information so that the robot can communicate a finite number of information to the other robot. Moreover it has to be defined a *code* thanks to which the other robot will interpret the signal and generate an answer. Let us suppose for instance that we decide to subdivide the water level information in only two possible values, and elaborate a three signs code which has to carry a two options information. This means that the possible values received from the first robot will be only 0 or 1, and the possible answer will be composed by the three possible values A, B, C. This way we will obtain a first stream of information: the first robot send a value and the second one generates other values depending from the received one. The code is constituted by the possible operation that the robot can undertake to express the message, the more they are, the more the message will limit the natural *equal-probability* of the elements. This means that the code, posing some rules, determines what can be considered an information and what only *noise*, what does a particular information mean and so on. Of course the more the code is complex, the more there will be possible to elaborate accurate information; the more the code rules will be strict, the more it will occur an entropy reduction.

  But let us go back to the concept of information. Established this type of robots communications, we can talk about information signals. This means that the receiver will react only in one possible way to the signal received, it will always relate only a precise meaning to a precise stimulation. Of course we can not say that this is not communication, but we can not say that there is *signification*. All the possible meaning are defined by a binary code of interpretation, or in another words by opposition (is the signal A or is it *not A*, therefore my answer will be B or *not B* and so on). [2] In this situation therefore the *signifiant* defines the *signifié* into a strict denotative way. The sign corresponds exactly to its information.

  When we involve humans into the communication process, the situation gets more

[2] There are semantics theories which describe every language as possible to be described by the process of *opposition*; every sign differs by an oppositive element and thus we create this way a working code. Nevertheless I do not debate this here.

complex and we introduce the concept of sense and signification. In order to explain the concept of signification I have to introduce the triangle of Odgen and Richards (1923) and therefore move to the next couple of terms.

- **referent/reference.**

  Since human beings are not computer, whenever they elaborate concepts, they generate a brain imagine of the concept. Even if dog exists by themselves, the concept of dog is constituted by our idea/image of dog, according to we fit the dog we mean when we talk. Before to go on I want to briefly spend some words regarding the concept of sign. As said previously in chapter two, a sign (according to Saussure and Barthes) is always the union of *signifiant* and *signifié*. A sign is therefore a symbol used by the sender the communicate a meaning. Now, if the sign into robots communication, automatically involves a peculiar reaction; it does imply a conceptualization to the human receiver. It will thus need to create a *reference* to elaborate the sign, an imagine of the *signifié* which does not coincide with the thing itself. There is thereby signification all the times it is create a reference. Then the sign will signify the reference, it will denote a concept out from the thing itself. This of course does not occur into robots, it belongs to humans. Furthermore in signification they are possible to exists references without a referent. For example the word *unicorn* has a precise reference but it does not have a referent, since it does not exist in reality but it does in the language and therefore it signify. It is clear that the concept of reference is central to the signification process. The signification introduces the following couple of terms: denotative and connotative.

- **denotation/connotation.**

  The denotation can be described as the basic process in communication: we will say that a sign is denotative when it refers the a precise reference which is determined by the code itself.

  For example the sign *dog* denote the reference of the dog, and every English speaker must agree upon this. It is the code of English language that defines this relation, it is part of the rules that it implies. In other words the denotative quality of a message is the form that the code impose to it to be accepted in communication.

  The connotation on the other hand, involves a more complex process which is possible only when there is reference and signification. When we say *cane* (equivalent of dog) in Italian, we do mean dog but at the same time we can mean bad person, bad in singing bad in doing something. Thus we generate a further relation of meaning: we mean dog with a denotative communication, but at the same time we could mean *bad* into a connotative way. Furthermore, the connotation implies a relation between the reference of dog and the reference of badness. This concept pushes us towards the next couple.

- **syntagma/paradigm**

I here start quoting Eco [3]:

It is established a code assuming that who is communication would have a dictionary of given symbols, among which being able to choice to freely combine following certain rules. We establish this way a kind of skeleton of every code, which can be represented by two axis, one vertical and one horizontal which are the axis of ***paradigm*** *and* ***syntagma****. The axis of paradigm is the axis of the dictionary of symbols and rules; the axis of syntagma is the axis of the possible combinations of symbols into syntagmatics unities more and more complex all the time, which constitute the speech itself.*

In other words, the syntagma are the logic part of the speech, the unities we put together in order to generate the horizontal developing of the speech. They are determined by rules and constitute the actual speaking process.

The paradigm on the other hand gathers together all the possible connotative options to vary syntagma. For example when we want to communicate the reference of: *He is a bad person.* We have obviously many diachronic possibilities for every word and group of words to denote the same message. For example we could say: *He is a cane*; or *He is mean*; or *He is sour* and so on. Paradigm introduces the concept of metaphor opposed to the metonymy; we associate two signs trough connotation or we substitute two terms trough connotation (syntagma substitution).

I want to conclude this little semiological thesaurus with a quotation and a chart of Jakobson (1963).

First I quote again Eco regarding an issue it will be treated also later in this chapter; i.e. the peculiar characteristic of music language where *signifiant* and *signifié* coincide into a process of signification and therefore with reference and connotative communication (of course they do not coincide as in robots communication, music does not carry signal information, but sense signs).

From Eco:

[...] the real source of information would be the code itself. The code as system of probability post over the equal-probability of the source, but in turn posed as a system of equal-probability compared to the huge series of messages that it allows.

I do not comment it here, it will come out later in the text.

Nevertheless, now I propose the Jakobson chart, in which he classify all the possible message functions:

- *referential*: the message denote a real reference.
- *emotive*: the message aims to generate emotions.

[3] Personally translated from: Umberto Eco. *La struttura assente* , pp.93-94

- *imperative*: the message represent an order.
- *of contact*: the message aims to generate a sensation in order to verify the communication.
- *aesthetic*: the message involves an aesthetic function when it is presented as ambiguous in its structure and it appears *self-referencial*, when it claims all the focus on its form.

I now leave this little semantics thesaurus and I go further with the discussion of music as language.

3.2 Music as language, levels of structure and syntax. Goal of the research.

The main goal of this research founds its roots in the human need to create new systems and contexts of sense, thanks to which the combining of music pitches is meaningful and musical phrases are possible to be created and understood by a consequential bond.

As stated in the previous chapter, I want here to approach a consideration of music language on the structural and syntactic level, not on the semantic one (I mean that even if I debate upon the issue regarding the semantic properties of music, it is relevant for the research the study of its syntactic properties). I will here study the structure of the music language, its code, how it is organized into grammar and how it creates rules concerning the developing of the speech. As explained, from others, by Volli [4], the studies on semiotics have certainly contributed to the overtaking of a stiff conception of music as *language without contents* taking back the debate to the idea of music as poetics. There is no doubt that music *means* something, even thought it is not necessary to refer to extra-musical contents.

The starting point of this work is therefore that music is a way of communication; that music is a language [5]. I want to remark again that this work deals with a specific aspect of music which gives not a complete overview on the whole music issues. This way of looking upon music is limited to tonal derived music, this work does not want to explain all possible musical approaches.

As stated in the room of reference, the discussion regarding the semantic faculty of music is referred exclusively to the tonal-derived music. The purpose of this work is definitely not to undertake an analysis of *music* in general, trying to explain it through semiotics. Nevertheless I have to say that I was tempted by these thoughts; it could be very challenging and interesting to generally analyze music language comparing it to linguistic and painting. Goodman in *I linguaggi dell'arte.* [6] has for instance proposed a study of notation systems analyzing the semiotic and linguistic properties of verbal language as well as *notational* languages as he defines music notation.

[4] Ugo Volli. *Manuale di semiotica*, p. 267.
[5] see, as example, Leonard Bernstein. *The Unanswered Question: Six Talks at Harvard.*
[6] Nelson Goodman. *I linguaggi dell'arte.* Milano: Il Saggiatore, 2013

I quote from him [7]:

A music score is a character of a notational system. Also in music notation not every character is a score, but I consider as score every character which can carry congruents. This excludes characters which are purely syncathegoremathics, for example, without demanding a score to be a complete composition or which is actually not-vacant [...]

And again[8]:

Standard music notation offers a familiar and significative example. It is at the same time complex, practical and - as the arabic number notation - common to different verbal languages speakers. [...] The multitude and the vigor of the recent riots against it shows the authority it has achieved.

By the way the comparison between music and linguistic in this case is useful exclusively to approach a definition of tonality which I start from in the developing of the method. I just want to remark that quarter tone music (and of course generally music) has not to be ruled by tonal rules; however it is ruled by them in this particular research.

Furthermore, I also want to underline that *I do not think that music comes compulsory out only from arbitrary precise rules, it can comes first the music and then its formalization into a rules frame.*

This assertion is meant to clarify my position in the debate between Darmstad structuralism and John Cage aleatory composition. Even if the Darmstad school has faced the issue of aleatory components in music composition, we can argue that integral serialism of *Boulez* perpetuates the tradition of occidental European music (from flemish polyphony of *Dufay* to Arnold Shönberg) which gives no space to random processes into the overall structure.

Structuralism proposes a definition, a conception of music possible to be divided into parameters; controlled, processed and generally elaborated into the composition. We could affirm that in structuralism, music can be described by a code which determines its characteristics before the composition itself. Saying it in a clearer way, music can be constructed by operations on parameters which always constitute it.

This aesthetic can be compared to Kandisky's abstractionism where the painting is disassembled into basic elements, later on combined and processed in order to create the picture.

I quote from *Punto, linea, superficie.* [9]:

[...] thing which should definitively take apart the fears towards a ***decomposition of art***

This approach to music lead us to assume that music can always be generated by itera-

[7] Personally translated from: Nelson Goodman. *I linguaggi dell'arte.* , pp.155
[8] Personally translated from: Nelson Goodman. *I linguaggi dell'arte.* , pp.157
[9] Personally translated from: Wassily Kandinsky. *Punto, linea, superficie.* , pp.11

tion of pre-determined processes; to assume that music can not be meaningful where there is no rules frame, where there is not a system, a code.

We can even trace back this stream bringing it to an epistemology debate that we can found already in Greek philosophy. From one side the Eraclitean *Panta rei* where the only realty is the continuous dialectic process; from the other said the *Hyperuraniun*, the *world of the ideas* of Platone in which there are set the paradigms of the things we can identify, recognize in reality.

If we assume that there are some base geometry in music, some basic items which constitute the music, we may array from the *Hyperuraniun* and affirm that music has some properties which do not belong to the *sensible experience* of the actual execution of itself. We are assuming that music somehow exist even before its duration in time, before to be heard. We are assuming, being close to artist such as Arvo Pärt [10], that there is some structure in composition that is not depending from the performance. We are somehow also arraying ourselves to the theory of *music of the spheres*, according to music some link to ideal geometries, giving to music some ontological value independently the produced sound.

On the other hand, if we array from the *Panta rei*, we are denying to music any immortal schemas, we are giving the right to be music to the only ones which has actually became sound and later on joined and interpreted by a listener. We are assuming that music is primarily *sound* and not music scores, we are according supremacy to audio files to harmony similarities between tunes and we are considering as completely different two pieces which can be written the same even though the sound completely different.

Of course this debate seems to be endless, since we can from time to time agree with both these positions analyzing reality, and music as part of it.

Approaching the studies of semiology and linguistics, I want here to quote a contribute to this debate again from Roland Barthes in his *Elementi di Semiologia* [11]:

[...] it has been this way elaborated the nowadays classic opposition between event and structure, opposition which has been peculiarly investigated in history. [...] Finally, that the un-conscious aspect of the language in whose which found in it their ***parole****, explicitly postulated by Saussure, appears again in one of the most original and fruitful opinion of Levi-Strauss, according whom the more un-conscious is not the* ***contents*** *(critique to Jung archetypes) ma the* ***forms*** *namely the symbolic function.*

And again:

This idea is close to Lacan who thinks that the desire itself is articulated as a system of signification, what induce or will induce to describe in a new the collective imaginary, comes not around its thematic, as it was until now, but around its ***forms*** *and its functions or the*

[10] Arvo Pärt. *Conversazioni con Enzo Restagno.* Milano, Il Saggiatore, 2016
[11] Personally translated from: Roland Barthes. *Elementi di semiologia*, pp.16-17

*be clearer: around its **signifiant** rather than around its **signifié**.*

To conclude:

*[...] We would thus postulate the existence of a general category of the dichotomy **Language/Parole**, extended to all the system of signification ; we will therefore apply these reasonings also to forms of communication in which the content is not verbal.*

This argumentations seems therefore to assume that, even if the debate has occurred for long times and still is occurring, we always perceive the inner structure of languages. We do share a common structure that allows us to de-code the messages and therefore we all share a base syntactic un-conscious knowledge that allows us to interpret all languages; music included.

Therefore when I pose the critique around the lack of universality of music language, I might go back to the ideas of Barthes, Lacan, Levi-Strauss and of course Chomsky; arguing that there actually exists a shared basic structure knowledge among all the languages [12].

Going back to music, on the other side from integral serialism, John Cage aleatory composition is proposing us a music which is not controlled by its birth, a music which is randomly generated, where the listener is defining an interpretation frame *aprés-coupe*.

Comparing it again to paintings, I want here to quote Deleuze writing on Francis Bacon [13]:

*What is a pictorial act? Bacon describes it this way: doing **random signs** [...]; throwing colours from different angles with different speeds.*

To Bacon these random acts (meaning acts which are not determined by a rational action according to the code of the language) are necessary to introduce the *logic of sensation*, they are necessary to push art forward introducing *human expressiveness* over the *clichés* of the code (saying this he is recalling Wittengstein with his *possibilities of fact*).

There is therefore an open and interesting debate between these two positions: can art (and subsequently music) be conceived totally before its concretization in a piece? Can we formalize art code after the interpretation of a piece (*aprés-coupe*) or does art carry always some structural elements we can not escape and we always use to generate and interpret it (*avant-coupe*)?

Personally I am torn myself between these positions.

If I can observe some elements always occurring in music, I can not accept that music can not

[12] I want here to remind that Barthes in his introduction to *Element de semiologie* assumes that semiology belong to linguistic. This means that, according to him, every system of signification has a deep connection with linguistics and therefore with language structure and its syntactic organization.

[13] Personally translated from: Gilles Deleuze. *Logica della sensazione.* Macerata: Quodlibet, 2004, pp. 167.

be born from intuition, sensation, inspiration. Even if I have undertaken the artistic choice to rely to tonality, I still believe that music (considering it with a global overlook) always involves inspiration and (especially through improvisation) it is determined also by decisions ruled by *instinct* and *feeling of something*. This means that even though I strongly rely on structures in composition, I always adopt the *logic of sensation* when it comes performance and improvisation. It means that I do believe that there are forms beyond the music, but at the same time they would be empty without the sensation which is set outside them.

I place myself near the ideas of Cezanne when he says: [14] *the geometry is the skeleton and the colour is sensation, colouring sensation.*

And again: [15] *it is mainly from the chaos where persistent geometry comes out from, the geological lines; and this geometry or geology has itself to pass through the catastrophe, so that colours could emerge and the Earth would raise itself to the Sun.*

Whenever there is music, there will always be some recognizable schemas which belong to music structure itself, even in random generated music. At the same time music has to experience *sensation*, *irrationality* to get colours, to become meaningful.

By the way, in this research I will study these geometries, these structures which belong to music language, I believe, deeply.

Moving on from this reflection, I want to point out that in this work the practice of music (to compose it and to play it) is central. The analysis of the syntactic schemas which constitute the music code and then a contest of meaning, is *the starting point of the research, not its goal.* [16] Also, I say it again: whenever I talk about sense, language and communication I refer them exclusively to the music form. This *sense* belongs to the form itself, not to elements coming or suggested from it.

Whenever it is created a system, and then a context of meaning, it is possible to interpret elements relating to it; it is possible to say what becomes coherent to it and what does not.

This is a fundamental structural consideration for every notational system; if we decide that an arbitrary symbol has an arbitrary property, this one will act on the other symbols which relate to this property; thereby those related symbols will change their meaning whenever the property has been applied.

The definition of a centre, a *key of reading*, along with the definition of rules of grammar, are the starting point towards the functioning of a system of meaning (such as tonal system is).

[14] Personally translated from: Gilles Deleuze. *Logica della sensazione.* , pp. 79.

[15] Personally translated from: Gilles Deleuze. *Logica della sensazione.* , pp. 79.

[16] Furthermore I do not try to compare music starting a structural analysis of music as language. Music can not come out from structure rules; they are useful to better understand the process of music communication and thus finally they are useful to define an harmonic method such as the one here proposed.

We can say the same for codes: random signs become coherent when we discover the correct way to interpret them; the correct way to relate them to each other according to a key of reading which we could define as a bunch of rules that imply actions and correlations between elements.

The syntax of music language is divided into *levels of structure* (as I named it), and organized by rules which constitute its grammar. As in verbal language, music has an alphabet which is used to construct words, with their rules of grammar. These words, mixed together, make sentences with their own sense, order and development. They will not exist words without letters, sentences without verbs and periods without complements. Music language similarly to the verbal one is ruled by laws which allow us to understand what concerns the topic or not, to define what sounds coherent or not. Without these rules, each letter and each word would be meaningless and they would not have any syntactic relation. This would make the message unclear for the one who reads. There would not be expectations or surprise: the development of the speech would be without a drive to any direction, in other words it would be flat.

As stated before, in tonal music (it has to be clear to the reader that whenever I talk about music I refer to tonal-derived music. I use just the word music the make the writing more fluent), after a dominant chord the audience will expect a resolution. This rule gives expectation and then allows to consider music similar to verbal language, with a syntax that rule the developing of the speech.

We can place a final reflection upon the codes and the syntax in music. Jean-Pierre Changeux, an important neuropsychiatry affirms in *I neuroni magici* [17] that:

Due to the existence of a connection between parts and the whole into an art piece, the beginning of a composition, a melodic fragment such as the beginning of a phrase, generates expectation of accomplishment of the meaning of the phrase. If this does not occur, or it does not occur in the appropriate way, we say that this accomplishment is uncorrect and it appears a particular wave in the electroencephalogram

This assertion seems to confirm that, even from a biological standpoint, music is built upon expectations and therefore it involves a code and a de-coding process (which according to the just exposed reasoning, could go wrong).

The issue is thus now shifted to the following question: " is this code belonging to music language or is it to a particular musical aesthetic?" Or in other words: "is it enclosure into the aesthetical *idioletto* or is it depending by the skeleton of musical communication?"

I quote Boulez from the same book:

There is not natural codes. Some codes just exists and they all are artificial [...]

[17] Boulez, Changeux, Manoury. *I neuroni magici*

Boulez clearly arrays to the position of a lack of semantic structure of music language, according to the composer the right to invent its own code to organize music. Even if I agree with this position towards a general consideration of music; I do believe that it exists a code we can not escape to perceive and therefore deal with into pitch organization.

Again from Boulez:

Which audience would rush to listen a Guillaume de Machaut Messe, which is is a very beautiful piece, but whom language could appear hermetic to whose is not used to the language of yesterday or the day before? A lot got enchanted by whatever Baroque music, without even being able to understand if it is beautiful or not.

As the reader can observe, here Boulez approaches music as a global phenomena. He is not limiting his reflections to the strict ambit of basic harmonic boundaries as we do here. Furthermore he is comparing Mauchant to an hypothetical Haendel, he is not comparing Mauchant to Alban Berg. Even if pre-tonal music does not follow strict rules as the Baroque music, we can easily affirm that they share the same syntactic basic rules: something that post dodecaphony does not (even notes are there discussed).

Thereby I do not want in this research to investigate *what is art* in music; I just want to look with a microscope the way music syntax is built, and explain scales with a mathematical formalization where notes are used and limited, in order to finally define a method to expand tonality with quarter tones.

Furthermore I do not affirm neither that syntax is compulsory to be followed in order to obtain a good result, neither that it regards bigger form, timbers and so on. *Au contraire* a deviation from the rule (as already pointed out previously) will generate meaning.

But in order to accept this, we have to accept the existence of some language syntax rule on the base. Whenever a new system to organize tones is created,it is posed above those syntax rules, it is a further class organization.

Furthermore, since the notes are twelve the permutations are limited and thus whenever we will hear a new code of tone organization, we will compare it to all the other possible ones that we already know. It is exactly thanks to this dialectic that we perceive news of contents, but we always have to have in mind the existence of these basic syntactic rules.

After this last reflection, I start here to analyze music syntax through the concept of levels structure. I explain what belongs to those levels, how they are organized and the properties they attend.

3.3 Level 0: the alphabet.

The first level of structure, named Level 0, is constituted by basic elements which can be combined in order to generate more complex elements. This creation process involves these basic elements which, combined through *permutation*, generate new elements. These elements are ordered, they can be inscribed into a series. They are equivalent but consequential and then equidistant.

Again, all the objects are made up of elements coming from level 0. More complex elements are generated by combinations of these basic elements.

In Level 0 there are placed the equivalent of letters in music: the notes. As I explained previously, many languages have their own alphabet. Some languages use symbols which mean already a definite concept (as example the Chinese language [18]); other languages use letters, that have not a meaning themselves, which are combined in order to create meaningful words. Music language is one of those: it uses letters, it combines them to create words and phrases.

Nowadays, Occidental music uses especially a system of twelve notes, when notes are used. By the way, there are other systems to organize music which do not use music notes (they organize sounds through other parameters).

For instance, recalling Boulez and the ICRAM, a lot of electronic and digital music has been conceived without definite pitches, or anyway without notes as traditionally intended in music (of course every sound has a definite pitch; however this does not mean that it has been made a selection of notes into the infinite possible divisors of frequencies).

However the reason which has guided me towards the decision to investigate the issue regarding pitch, harmony and scale can be found in a personal (or maybe not really personal) problem (already partially approached before).

I have to say that many time I was attracted by the avantgarde and I therefore tried many time to approach *la musique concréte.* I was pretty often tempted to approach the composition of music which does not use notes, a music made up of *sounds*, timbers and rhythm. However anytime I tried to avoid the issue regarding the pitch I got stuck into it. Surely every sound in nature has timber, duration in time (rhythm feature), intensity (dynamic) and frequency (pitch). The reader might then argue that whenever two or more sounds are put in relation, automatically it has been generated a relation between two pitches. Thus, even if I tried many time to not organize sound according to some *note-oriented* schema, my ears always claimed the relation to some of them. I could not avoid to hear intervals and therefore to interpret them as part of a scale, tonality, interval area, twelve-tone series and so on.

[18] In the written Chinese language, the *simple* characters are pictograms, direct representation of concrete objects such as sun or moon, or ideograms, representation of ideas or abstract concepts, such as up or down. Not all the Chinese written characters are of this type.

This personal issue, faced into the practice of composition and improvisation, drove me the convincement that there actually exists a structural code in music language which belongs to pitches and their relation through notes.

Therefore, if the reader is questioning why am I insisting that much on the syntactic analysis of music, I answer saying that surely it exists music which does not intend notes as in tonal, modal or serial music; nevertheless I believe we can not deny the tendency of our musical ear to perceive relation links between pitches.

Going back to the discussion on the musical alphabet, these, previously mentioned twelve notes, constitute the chromatic scale which can be compared to the letter alphabet. Every music, if there is notes, makes use of these twelve notes: there is no other possible notes, but many possible combinations and key of reading as well. As it is in the verbal language, alphabet is a tiny limited series of symbols compared to the list of words we can find in a thesaurus. However all those words are made up from those letters and no others. Furthermore those words has a meaning themselves but they acquire other meanings when they are combined through syntax. There are more levels of meaning that coexist, lots of possible combinations of letters but a limited alphabet. It is the same in music, there is a limited number of notes but there are much more scales, intervals, tonality, chords and so on.

I introduce now a formalization of the notes alphabet that approaches an additional process. This can be considered my bridge to cross over the river of words and approach the mountain of math and geometries which (as stated before) takes an important role into this research.

These twelve notes (C,C♯, D, D♯, E,F, F♯, G, G♯, A, A♯, B) are the same important, but each one fixed higher-pitched compared to the previous one up to reaching the octave (double frequency from the starting point). Consequently it is relevant only the distance between the notes (called interval or relative distance).

The chromatic system consists of elements that we can call, as example *A, B, C, D, E, F, G, H, I, L, M, N* in which *B=A+1*; *C=B+1* et cetera, where in twelve tones music the value 1 is called half step and the value 2 whole step. All the remaining distances are obtained through the combination of halftones, and they are named specifically according to the distance between and the name of the notes.

For example: C-Eb is a minor third (three halftones); C-D♯ is a second augmented. Due to the name of notes they are named differently but both are three halftones.

The structure of a tonal system in music is based on the study of a set of notes characterized by their relative distances into an octave.

My research starts within the Level 0.

In nature, sounds are made up by sound waves. Every sound has its own pitch, timber, intensity and duration in time. Pitch is measured in Hertz (since it is a frequency) and is a

continuous value. Continuous values are infinite divisible. Therefore in order to create letters to fill our level 0, it has to be defined discrete values of pitch.

I said before that there is a lot of possible combinations of letters which creates words, by the way in music the permutation process has to fit some more strict rules. [19] This is why I decided to expand the alphabet from twelve up to twenty-fours notes: so many has already been researched upon the permutation of twelve notes that I decided to act already in level 0 in order to quench my thirst of new paths. [20]

[19] In verbal language ideally there is no rules to create words from letters except the phonology and social determination of the language. However we could imagine an imaginary language in which some words are arbitrarily defined starting from an arbitrary alphabet. They will be possible to carry the meaning we decide to assign them, no matter how they are constructed.

This is possible since in verbal language we divide the semantic content of symbols from their phonetic content: we divide the sound from the idea, the *signifié* and the *signifiant*.

In music of course we can not separate sound and meaning since the sound is itself the meaning. There is not *signifiant* in music, we may affirm that content and shape coincide.

We may assume that music can be considered as the language of form. Music (as artistic language and therefore possibly together with other artistic languages such as paintings, sculpture and so on) gives forms a meaning, it accords to the *signifiant* a semantic meaning. There is no sign, the sign coincide with its semantic content. This is why Nelson Goodman refers notational systems to the music notation (which uses symbols instead, graphic symbols to refer to sounds) and claims an analog system for the music language itself. Since there is this perfect juxtaposition of sign and meaning we can assume that there is not needed a code to interpret signs which refer to a meaning out of themselves.

Au contraire there is a direct stream of information. Especially in music there is even less representation than in paintings. If we can recognize a horse and a bull in *Guernica* we can not say the same for the fifth symphony of Beethoven. The bull and the horse of Picasso are of course not notational signs, they are not *signifiant* of the *signifié* of horse and bull, but they anyway carry some memories, some dross of representation (this is exactly what Kandisky, Cezanne and Bacon wants to avoid. This is probably *the issue* of figurative arts: to release language of figurative arts from the illustrative dross.).

This might not drive us towards the conviction that music do not carry any precise content. Simply this content is into the structure of music itself: the way that music is constituted contributes to define the *signifié* of music.

[20] Going back to the previous footnote, since music structures (or music rules) contribute to define music *signifié*, to create new system to organize music means to give music new contents. This means that to communicate with music implies to research new music systems.

We can easily observe this through music history and its study. The biggest part of book on music history divide music streams according to some harmonic or rhythmic developing of the music practice. The reader might very probably recall some books in which there were some sentences like: *They occurred some real battles around the usage of some chords and chords progressions, and it is clear that often the forbidden passages will often researched without a real necessity. They will be held simply as a statement, a proof of faith* (freely translated: from Massimo Mila. *Breve storia della musica.* Torino: Piccola Biblioteca Einaudi, 1963 , pp.207.).

It is not too risky to assume therefore that the research upon music language structure can coincide itself with the need of the artist to communicate new contents. The communication of new contest passes through the usage of new forms.

This seems to confirm the previously exposed idea regarding the perfect collimation of *signifiant* and

The definition of the music notes has been an issue for centuries and, even though it is not the core of this research (we take for grants twenty-four quarter tones notes), I will spend some words upon it.

Historically the problem was to pass from a continuous measurement to a discrete one. [21] Starting from the pitch of a sound event it has to be found some pitches which are related to the fundamental. The choice of intervals is related to a fundamental sound (the word interval claims itself a relation between two elements) and that one is the base context where the notes in the alphabet has been created. In other words, the process towards the definition of notes has to start (again *keys of reading*) from on pitch. Taken that as a starting point they have to be chosen some other notes relating to it. This is the way it is possible to define some discrete values (definite notes) into the infinite divisible real numbers of the Hertz measurement. Recalling again a picture from the ancient Greek philosophy, it is like the paradox of Zeno: *Achilles and the tortoise.* I propose it here:

In a race, the quickest runner can never overtake the slowest, since the pursuer must first reach the point whence the pursued started, so that the slower must always hold a lead. - as recounted by Aristotle, Physics VI:9, 239b15 . [22]

This paradox is considered one the first argument regarding the infinitesimal calculation. Achilles will never reach the tortoise since it will always be a step further. This is possible since the line the are running on is infinitesimal divisible. It is the same for the frequencies values: the sound pitch can be infinitesimal divided.

Therefore in order to find some notes we have to put some leg, some milestone on the path. It has to be defined a limited division of the line, but how?

The first value taken from the infinite sound vibration frequencies, our first stop-over is the octave. The octave division can be found in every culture all over the world. [23] This

signifié in music; where **form is content, music structure is (at least partially) the content of the message.**

Furthermore the effort to interpret the message, the shape the ambiguity of the form appears to be the **main goal of the musical message**.

Quoting again Eco:

A productive ambiguity is the one that awake my attention and drives me to an interpretative effort, but which later on allows me to de-code the message , allows me to find in that apparent disorder an order [...]

[21] To examine in depth the question, I suggest this text:
Stuart Isacoff. *Temperamento, storia di un enigma musicale.* Torino: EDT, 2005.

[22] Wikipedia; *Zeno's paradoxes;* page last edited on 7 September 2017, at 05:43; Available at: https://en.wikipedia.org/wiki/Zeno%27s_paradoxes [Accessed 11 Sept. 2017].

[23] This is an important statement since according many ethnomusicologists the octave is the **universally recognized interval**, no other interval, no other note can be found in all the music across the world, only the octave. This is probably because the special mathematical properties it has since it is defined by a double frequency from the starting one.

division uses the ratio 2:1: the second sound has a double frequency compared to the first one.

Here I place an important consideration: the ratio 2:1 can be compared to the Arabic number base. As, when we count in base ten, every ten numbers the zero appears, so in music after the octave all the notes repeat with a *higher value.*

In base ten we only have ten number symbols (nine plus zero), after ten all the *symbols* repeat with the zero. They do not change their specific quality, they just come up to a higher level. The same is in music. In fact, the second and the ninth have the same function even though they are an octave distanced.

From that infinite range of frequencies, we obtain an initial division in discrete values: anytime that the frequency is in proportion of 2:1 compared to the original, we have the same type of sound on an *higher level.*

This allows us to start the research of discrete values *in between the octave* to create further divisions into the proportion 2:1 and finally find the remaining music notes.

In this research, the octave then generates the edges of our set. **All the notes, all the letters of music alphabet, appear into the range of the octave.**

As stated previously, since the octave division is universally recognized in all the human cultures, we do not skip it and we take it as limit of the set. [24]

Once the set of octave has been defined, we can propose different divisions in it.
Equal temperament proposes an equal division into twelve equidistant notes, pythagorean scale defines notes through geometric proportions, arabic Maqām propose a twenty-four equidistant division and so on. As the reader might have thought, there are many possibilities into the definition of the music alphabet elements.

3.3.1 Brief history of music alphabet.

The Pythagorean system is one of the first attempts to define music notes. The Pythagorean system (and the consequent Pythagorean scale) is an example of a process used to define the musical notes through geometric division of the wavelength. The approach he adopted to define divisions into an octave is surprisingly similar to our leg division of the Zeno's line. He physically divided a sounding string in portions [25], putting some leg on the line.

Pythagoras has adopted some craft and geometrical principle to find consonant sounds starting from the fundamental vibration. Since he believed into the supremacy of the *numbers*

[24] However it would be possible to conceive music that skips the octave. If we proceed by sixth of notes for instance we do not reach the ratio 2:1. Some music has been produced skipping the octave, but anyway I am not considering it in this research.

[25] Actually it was an iron line into a blacksmith; the legend tells that he was passing by when he heard consonances between sounds coming from a blacksmith laboratory. He thought it sounded like bells and he wondered how was it possible that there was that consonance. Therefore he entered and started to reflect upon the music issue, he tried to solve the misery with his typical mathematic and geometrical approach.

into the construction of reality, he tried to find some basic proportions to defines consequent sounds.

The first division was of course the simplest proportion, 2:1. [26]
After this division he did not consider the octave as edges of set in which go on with further division, *au contraire* he decided to start again from the fundamental sound trying out the other simplest proportions. So did he and therefore he divided it in $\frac{2}{3}$, getting so an interval of right fifth and, reversing it, right fourth [27].

The reiteration of fifths generates the notes of our major scale excluding the fourth degree (which we obtain going a fifth backward from the octave). Pythagoras obtained this way a scale very close to our major scale. Anyway the usage of perfect proportions (which will start the concept of *Musica universalis* in the Middle age all along through the Renaissance) do not generate equal half steps since the reiteration of fifths do not go back to the starting point but it shifts slightly. The issue of the equal temperament has been debated harshly for very long time involving icons such as Galileo, Newton and Rousseau.

The scale we adopt nowadays was now easy to achieve and it is still object of discussion.

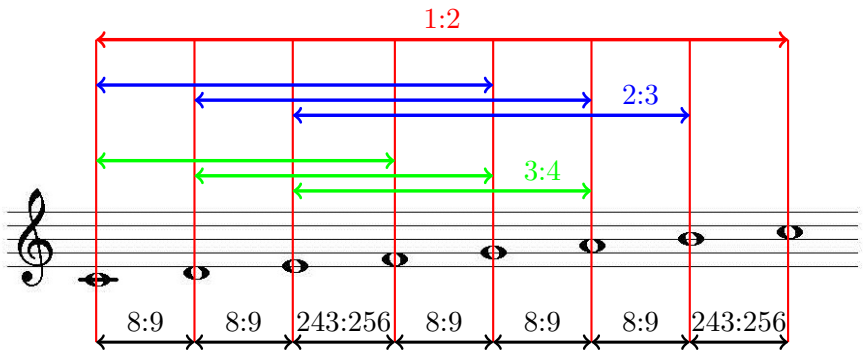


Figure 3.1: The Pythagorean scale

Jumping on further in time and into a parenthesis, I would like to introduce the notion of natural harmonics.

We know now, after scientific studies upon the physics of sound, that any kind of vibration generates at the same time several vibrations with different intensity and decay. These vibrations are in fact called overtones and they appear into a series which is called the series

[26]The octave division can also be found in every music culture all over the world. I quote here Patel from, Aniruddh Patel *Music, language and the brain.* Oxford University press, 2008, p.13:

Such octave equivalence is one of the few aspects of music that is virtually universal: Most cultures recognize the similarity of musical pitches separated by an octave, and even novice listeners show sensitivity to this relationship.

[27]Please note that the fourth is also the inversion of the fifth. Schönberg insists a lot on this relation between those two essential intervals

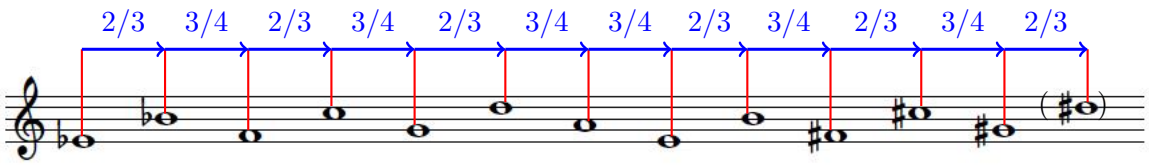


Figure 3.2: The chromatic scale

of natural harmonic.

Since it was possible to analyze the sound and the harmonic phenomena with a scientific eye, a lot of musical studies has been made upon it. Before the electronic and digital music, Schönberg [28] and Henry Cowell [29] for instance have studied the music development through the study of harmonic series. Especially Henry Cowell has proposed an analysis of the music evolution through the analysis of how musicians have time after time used harmonics sounds more and more further. He assumes that the evolution of the harmony and even of the rhythm [30] depends from the acceptance of notes coming from far overtones. When musicians have accepted the seventh as a consonant interval into a chord, then the nine, then the sharp eleventh; the harmony has grown up.

The choice of relevant notes is therefore depending (according to Cowell) from the natural series of harmonics which is incontrovertibly existing in every sound.

From the idealistical *Musica universalis* where the world is made according to the unquestionable wisdom of God (again the debate between *gestalt* and *matters of fact*), to music which is ruled by physics; imprecise proportions that rule the sound phenomena incontrovertibly. [31] The fifth and the octave (2:1 and 3:2 of pythagorean system) are the first and strongest sounds appearing into the harmonic series. The fifth (ratio 2.3) is then the second more universally adopted interval after the octave.

[28] Arnold Schönberg. *Trattato di armonia,* pp. 22-33

[29] Henry Cowell. *Nuove risorse musicali.* Milano: Ricordi, 1998, pp. 27-51.

[30] There is a very interesting chapter in *Nuove risorse musicali* in which Cowell proposes very complex polyrhythmic textures which come out as a translation of harmonic intervals into rhythms.

He proposes to ideally slow down the vibration of the strings which are vibrating in a consonant proportion according to the overtones. Then he imagines to slow the movement down until the interval becomes rhythm, polyrhythm.

[31] I want to call also Einstein to contribute to this debate between order and chaos quoting his famous phrase: *Gott würfelt nicht mit dem Universum.*

There is an incredible gap between the biggest and the smallest in physics theories. Newton laws are valid, the theory of relativity is valid as well as the *Quantus mechanics* in which probability rules the physics. How is possible that probability rules the destiny of the particles of the same matter that, became big, is ruled by absolutely precise rules?

Again we can now says if there is some schema or not. We can not say if there is a God, if we are that one or simply we too hysterical to live life without questioning it and trying to describe and somehow dominate it.

It seems therefore that there is some kind of connection between physics and musicians choices. It is very interesting to remark, although the fifth is unanimously recognized by all cultures, that some of them skip it. For example in Gamelan's Javanese music it is uses a pentatonic scale (Sléndro) which is the result of an octave's division in five equal sections (the fifth cannot be found when we divide the octave into five equal portions).

Going on analyzing the series of natural harmonics we now face the second octave (forth harmonic, or else 4 times of the original frequency), the third, the fifth, the minor seventh, the ninth, the eleventh augmented, the fifth, the minor sixth, the minor seventh, the major seventh and again the octave. We can observe that the strongest harmonics are the first

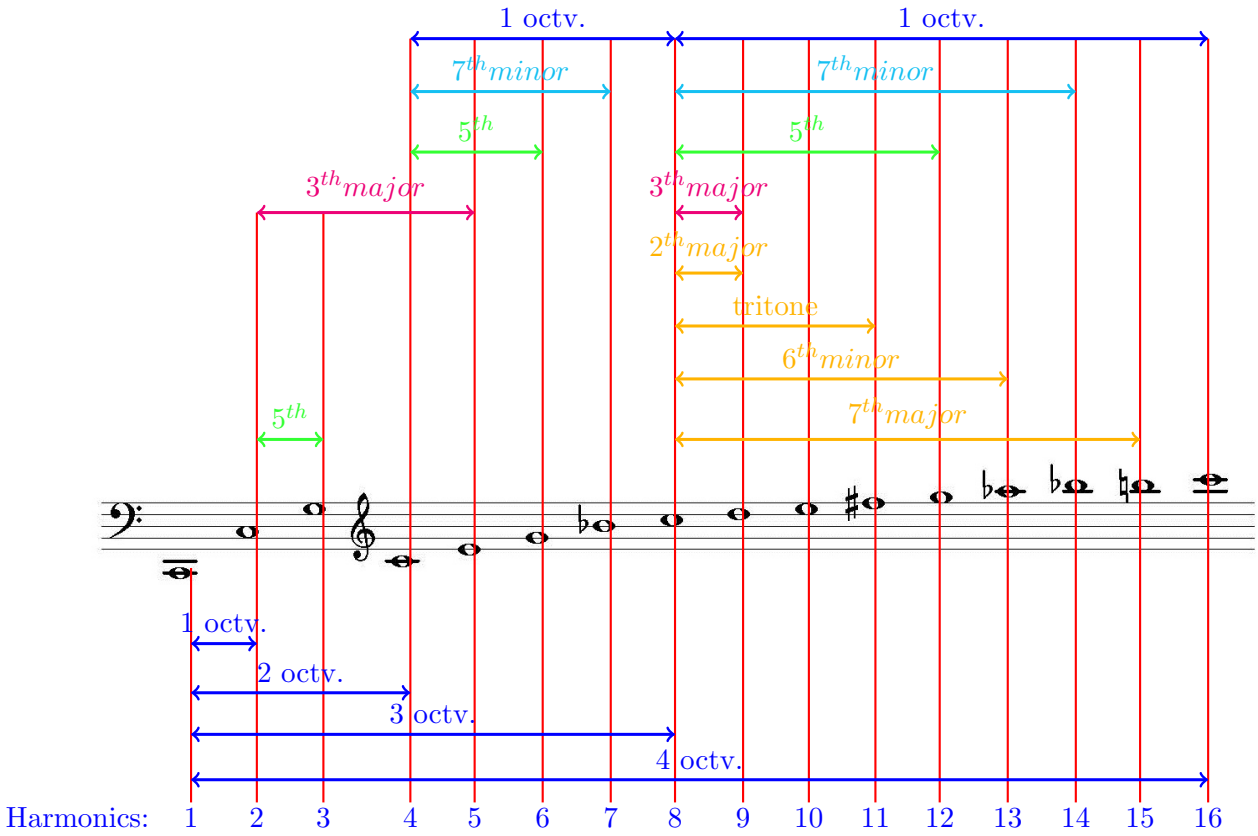


Figure 3.3: The harmonics

three, the octave, the fifth and the third. They occur the most into the overtones. We can therefore observe the *natural foundation of the major triad.*

I want the reader to observe that, the more we keep on dividing the octave, the more the different cultures have distinguished their music choosing some notes instead of others. In classical, Indian, African and Oriental music the choice is different.

Ending here our short journey among the history of temperament, I place this last reflection.

The different cultures among the world have found different needs to express their own music alphabet, but *they all have had the necessity (at some point) to limit continuous values to discrete ones.*

Not all the music uses definite pitches and music notes, however notes appear in every culture all along the world. [32] This drives us back to the previous reflections and convince us that music structure (starting from the choice of its alphabet)is a big part of the music content, of the music meaning. Different culture produces different music and therefore different systems whit different alphabets.

The fact that we can find the usage of notes in every culture all over the world may suggest us that there is some *universal grammar* (to use Chomsky' s words) which regards pitches. Every culture has found the need to produce a system that could have communicated meanings through notes. This confirms that there is a *gestalt* regarding the music pitches, it confirms that everyone can perceive structures into tones organization.

3.4 Level 1: music functions.

We now construct level 1 from level 0: from letters we approach words. Building words into the music language means to gather some notes together in order to give them a peculiar characteristic, it means to operate a choice of some notes putting them in relation. As already pointed out, the choice of several notes allows the creation of meaning systems which give order and sense to sounds.

To generate words means therefore create some schemas that are able to rely notes to each other and then to give them a property. This property belongs to these letters which, when put together according the schema, acquire a peculiar characteristic that defines them as a superior order object.

In other words: defining a point of reference these models create a schema which relates different objects and gives them a sense in a higher syntactical level. For its part, Schönberg in his *Harmonielehre* talks about the birth of the Occidental music alphabet and the tonal system development until its extreme periphrasis. He analyzes all the relations of sense, even the most far, which thus carries a more labile grammatical relations between objects. [33]

We can assume that there is also a multiple layer of notes organization in music. On one hand we can define some schemas which give syntactic sense to notes and which therefore define some harmonic properties of the notes. These schemas are the ones which allows to talk about tonal functions and *cadences*, they are the one which define scales and chords.

[32] I want to quote Aniruddh Patel again regarding this assertion, from *Music, language and the brain*, pp.21:

More interesting from a cognitive standpoint is the predominance of asymmetric scales. This suggest that most cultures favor scale patterns that promote a ***sense of orientation with respect to a tonal centre****, and raises the deeper question of why this should be.*

[33] Arnold Schönberg. *Trattato di armonia.*

On the other hand music notes are also put in relation according to phrases. Those phrases are often enclosure into a scale, into an harmonic frame which contextualize them. However they have a proper inner organization and *rasoin d'être* that can go along with the harmonic context or not. So much that we can recognize harmonic sense of a phrase as well as the phraseology itself: they are disjointed.

Furthermore, phraseology involves rhythm since it belongs to the developing in time of the lines. As in the spoken language, the rhythm of the speech is relevant and the prosody gives sense to the communication as well as the words that are spoken.

We could consider words in music (or more neutrally objects belonging to the level 1) both schemas which organize notes according to their harmonic properties, both the ones which are doing that according to the phraseology.

In jazz improvisation for instance there is a consistent developing of phraseology towards improvisation. The soloist articulates *phrases* on a constant pulse which assumes the role of a quantization grid in which rhythms ideas emerge in a really efficient way.

Furthermore the soloist has to build his or her own *language* similarly to the spoken one. The approach towards the rhythm conception in jazz phrasing recalls the way we create rhythm with words. In fact there are lots of similarities and links between jazz and rap music, since they both involve the stress on rhythmic ideas on the top of a constant beat.

However, since phraseology belongs primarily to rhythm, in this research it is relevant for me to approach the schemas which are relating notes according to harmony rules; the ones which are able to generate harmony boundaries. Phraseology might be object of further studies, for the moment it is important to understand how to create systems which are possible to allow harmonic connections, *cadences*, grammar into a twenty-four letters alphabet (this aspect will be deepen in chapter 5.2).

I go on approaching the issue of harmony generator schemas into level 1.

The occidental tonal system is a concrete example of linguistic system of interpretation from a determined number of elements. Again, in level 0 all the elements are equivalent, there is no hierarchy and the possible interpretations of the notes are available but yet not defined. *When we set one note as core, as key of reading, then all the others acquire a meaning relating to each other into a net.* Therefore, if before C, C♯, D, D♯ were all equidistant and independent, now C♯ becomes the minor second to C and D the major second and D♯ the minor third.

From a mathematical point of view, the equidistance and equality between all notes in level 0 makes possible the application of systems always repeatable starting from any of these elements. This is a peculiar aspect of a tempered system, probably the one because it was chosen among other temperaments. Since all notes are equal before we set a key of reading, we could repeat a schema that we have applied one note, exactly on another one and we will obtain the same relation translated.

Oppositely, according to the different context, the system's elements have several roles each time different. The same sounds will mean something completely different when they will be gather according the a schema or to another. For example: the same note A can be considered as the *tonic* if we are in A major; but it would be the major third, the *mediantic* if we are in F major.

Placed this first step, we have the opportunity to think of several series of relation's schemes between the elements, each one with an unique identity.

As it will be explained in details in chapter 4.1, the creation of a scale (first passage to build the tonality) consists into the definition of an arbitrary series of distances in order to assign, using an additive process, the note values of a scale starting from one note.

For example if we define the function *major scale* as:
f(a)= [a; a+2; a+4; a+5; a+7; a+9; a+11; a +12]
Then: f(C)= [C; C+2; C+4; C+5; C+7; C+9; C+11; C +12]

If we operate into a twelve tones alphabet, in level 0 the smallest interval is the half tone, so that 1 is halftone, 2 a tone and so on. [34]

Please note that the total addition of elements always has to be due to octave. That means that in this example after a+11 will come a+12, an octave higher. This concept will be explained better in the next chapter where a mathematical formalization is proposed. However we say already now that this operation is called *modular operation*, since all the additive functions we define are closed to the octave, that is a finite set of elements (so in this meaning, the codomain of the tonality function is finite).

If we calculated f(E) instead of f(C), we would have had the same distances, but we would approach new tonalities. The function *major scale* will always be the same to itself but it will generate different notes when we to chose a different independent variable. When we will hear a major scale of C and a major scale of E we will then be able to recognize the same relation system, being anyway aware of the difference between the notes that have been used for the construction of the scale.

Every scale is related to a specific tonal function and viceversa.
Pentatonic scales, blues scales, major scales, minor melodic and minor harmonic scales, each of those have a particular identity and sonority therefore each of those is related to a certain function and so a different model with its own internal rules.

We can therefore assume that objects belonging to level 1 are *schemas of relations*; they operate on the top of the notes generating objects which are recognizable besides the notes

[34] We can easily translate [C+2; C+4; C+5 ...] in [C+1T; C+2T; C+(2T+1/2)] obtaining a series T,T,Ht,T,T,T,Ht or [a+T; (a+T)+T; (a+T+T)+Ht ...].

themselves. [35]

Our perception is possible to understand this double layer of information in juxtaposition: the notes (which note is it this one? Which tonality? Some tonalities are brighter than others and so on) and the relations on their top (is it a lydian dominant context? Had that group of notes had the function of sub-dominant?).

As the reader can observe there are always multiple levels of information occurring in the music language. We can already identify three levels of contents (notes/pitch; harmonic meaning of lines; phraseology) in level 1, regarding the organization of pitches. There is more into notes organization as well as rhythm, timbers, dynamics... Music language is therefore very complex and it operates always on multiple levels of meaning, every one juxtaposed to the other, independently one from the other but possible to be in relation as well.

This implies a further complication from the *polysemic* property of music language: even in one semantic ambit there multiple levels, multiple layers of meaning.

This way we have thus defined the objects belonging to the level 1. If previously we carried on finding the notes, now we establish many functions which allow us to organize them creating multiple levels of meaning according to a centre (or given core point, again the *key of reading*).

Whenever we will hear a G and an A in a row, our ear will try to interpret this series according to one of the schemas constructed into level 1, *referring to a scale*. The attempt of our ear to place melodic fragments into systems of meaning put on highlights the capacity of tonal functions to generate harmonic function into a linear (horizontal, not chords) developing of the notes.

Going on with the formalization, together with the additive tonal function we associate a further organization of notes which assigns to every note (obtained with this process) one of the three possible meaning values: Tonic, Dominant and Subdominant. [36] These three values consist of the Tonal areas, in which all the notes and chords of a scale are inscribed.

These three areas are recognized by listeners same as verbal language predicates. They allow movement and logical consequence into music development. The Dominant area has an unstable and propulsive nature which tense to solve into the area of Tonic (stable, which define the center of the musical speech) while the Subdominant area has the function of preparation for the Dominant and it is antithetical to the Tonic. These areas create expectations in the horizontal developing of the tune, defining a grammar of the musical speech. Of course, it will anyway be possible any type of movement, but whenever a movement will be different from the ones described by the rules of tonal functions, our ear will perceive an

[35] Please note that all these reasoning along with this mathematical formalization, approaches music from the aspect of possible combinatory patterns into a limited set of notes. All the other aspects are left over, it is here relevant only the possible combinations of schemas between elements.

[36] this is valuable only into tonal systems. For example in Indian music there are some scales which we also meet in European music but since they are not supported by a tonal context their notes cannot be inscribed into tonal areas.

unexpected evolving of the speech. [37]

The concept of tonal areas and cadence (movement between tonal areas) is therefore fundamental for the musical syntax. It actually is the seed of it, it defines the rules of movements and congruence.

Our previous function *major scale* is now decorated with the tonal areas:
f(a)= [a; a+2; a+4; a+5; a+7; a+9; a+11; a +12]
where [a; a+4; a+9] ∈ T ; [a+4; a+7] ∈ S; [a+7; a+11] ∈ D

In which the order relations (or lines of order) are S-D-T (Subdominant - Dominant - Tonic); which means that possible movements should be: S-D-T; S-T(Plagal cadence); D-T(Authentic cadence); S-D(Second-Fifth).

Example in C: [38]
f(C)= [C; C+2 (D); C+4 (E); C+5 (F); C+7 (G); C+9 (A); C+11 (B); C +12 (C)]
Tonal areas:
{C+9 (A), C, C+4 (E)} ∈ T;
{C+2 (D); C+7 (F)} ∈ S;
{C+7 (G); C+11 (B)} ∈ D;

As previously highlighted, the tonality obviously never excludes the twelve notes but simply orders them through the definition of some preferred consequentiality links and neighborhood links between notes. The notes which are external to the function become in relationship with the system as *out* of the system notes.

Now we can talk of musical grammar: a chosen group of ordered notes has a precise meaning referring the context. The organization of music is made (starting from a chosen note) by the definition of other notes (according to the usage of musical functions) and by putting them into the three possible areas of meaning S - D - T.

Now it should be clear that, once more time: ***this step is fundamental to the creation of a system of meaning.***

[37] I made this clarification because I do not want the reader to think that I propose an analysis of music where there are precise rules to the good composition.

There are rather some rules of system which allow the listener to understand the will of the composer.

[38] Into this scheme of division between Subdominant-Dominant-Tonal, we can recognize the seed of the thirds constructed chords:

- T= [A, C, E] namely [C-2 notes; C; Do+2 notes]
- S= [D, F] namely [D; D+2 notes]
- D= [G, B] namely [G; G+2 notes]

Where +2 notes means, having a look to the seven notes scale [a,b,c,d,e,f,g]: a+2 notes= c ; b+2 notes= d ; and so on.

3.4.1 Vertical functions, superior order objects, chords

Now I want to go briefly through vertical harmony; through chords.

Thanks to the usage of the thirds superimposition (according to the tonal areas rules) we obtain first the bi-chord then the triads, the tetrachord and so on. Why do we choose third intervals to build chords? Why even if we construct chords with fourths (as in the jazz harmony evolution from Bill Evans and especially Mc Coy Tyner up to Wayne Shorter) we always explain their functionality assigning them a correlative third constructed chord? [39]

The thirds superimposition founds itself on a constitutive principle of the tonal areas: the objects contained in one area are interchangeable (since they have the same function). This also means that they can be superimposed.

Since we have previously defined schemas of horizontal organization which assign to notes an harmonic value (or function), we can now gather all the objects that respect that property: we have created a *class*.

We have created a label, a property that belongs to more than one element and that allows us to put those elements together as those elements which respect that property. For example let us consider the class named *colour:blue*. Now, we can affirm that the ocean

[39] Chords made up of fourths, if they are used into a tonal or modal context, are commonly considered (especially into jazz praxis) particular *voicings* of some thirds derived chord. Let me propose an example. Let us consider this chord made up of fouths:

D - G - C - F

This tetrachord is all made up of perfect fourth. Nevertheless when we analyze it in order to get its harmonic function we re-construct it in this way:

D - F - C - G

Simply shifting the same notes into a different order, we obtain a third constructed chord with some omissions. We can observe that we have a D - minor third - minor seventh - eleventh.

As the reader might cleverly observe, whatever organization of twelve object through an interval will at the end go back to the same twelve notes, simply putting them into a different order. We would reach the same twelve (if we proceed chromatically) or seven (if we proceed diatonically) notes. Let me propose an example: I now construct an ideally endless thirds chord into the scale of D dorian.

D - F - A - C - E - G - B - (D)

As we can see, it has been reached the starting point, the octave.
I now proceed with fourths:

D - G - C - F - B - E - A - (D)

Same number of elements, same elements; different order.

The issue becomes therefore: why do we always rely to the third construction to define harmonic function?

as well as the sky and a blueberry are blue, thereby they can be gathered under the class *colour:blue*. The same with the notes.

Let us consider the context of the C major tonality: the tonal areas are Tonic (C; A; E); Sub-Dominant(D; F); Dominant(G; B). Now we can say that the notes C and E are both under the class *tonal area: Tonic* and the same for A and E. This means that if we are interested into the harmonic property of the notes, the notes C and E share exactly the same value, since they belong to the same class. This means that they can be superimposed and this also explain why the third construction leads the definition of tonal function into chords construction. Furthermore, as the reader who has studied harmony surely knows, the chords of Cmajor and Eminor are possible to be interchanged, again because they share the same harmonic class value.

With the chords construction, with the *vertical moment* of harmony, objects with a higher level of complexity are born.

These objects have their own internal vertical relationship even though they inscribe themselves into horizontal functions of level 1. The major triad, same as the major scale, will always be recognizable as a particular object itself independently from its starting note or context, but it belongs to scales which determine its tonal function and then its function into horizontal developing.

Again there is a coexistence, a juxtaposition of different levels of meaning: many classes are acting on the elements giving them plural values of meaning.

The major triad is built with the superimposition of a major third and a minor third:
Fv(a)=[a+4; a+7] [40].

Same as we said for scales, a D major triad will sound the same of a C major triad and an E major triad and so on. However, these objects of higher order are found through the thirds superimposition process into several horizontal systems. For example, the vertical function *major seventh tetrachord* can be found as the first degree of an horizontal function *major scale*, as a fourth degree of another *major scale*; as sixth degree if *minor harmonic*.

The object Fv(C)= [C; C+4 (E); C+7 (G); C+11 (B)]
can be found in C major (I grade), G major (IV grade) and E harmonic minor (VI grade).

This feature creates a double internal force into the object of higher level. As soon as the next object does not show up this Cmaj7 could belong to three different scale functions (one belonging the Tonic area, one to the Subdominant, one the minor Tonic) still keeping his own identity of major seventh chord.

Through these higher order objects, the syntax gets more complicated: they generate a net of possible relationships between horizontal systems (tonalities). They allow (starting

[40] here Fv means vertical function.

from a single object with multi-functional property) possible paths across horizontal systems. They can create (if used in a continuative way) a chromatic pan-tonality where the consequentiality bond between a horizontal function and the other go weaker.

Wagner's music for instance is an example of this process of elaboration of the syntax. As it happens with the *Tristan chord, F, B, D♯, and G♯*, the tonality gets more and more ambiguous and the relationship between tonal areas becomes less consequent (or predictable). Diminished chords and altered chords allow many different resolutions, they connect a lot of tonalities. Wagner uses a lot this property stretching the language grammar. Without chords (higher order objects) it would not been possible to stretch the syntax as Wagner started to do.

Furthermore, chords are useful to the ear to undertake an harmonic function analysis of the lines. This implies that we always perceive a connection between lines and chords, even if vertical harmony functions do not appear in music. This concept is very important towards the analysis of tonal areas movement into tonal scales. It is treated further in chapter 5.2.

In this research I will not study the chords into a twenty-four notes context; nevertheless it could be study of further researches and I believe that this parenthesis was really meaningful into this explanation of music as language.

To the level 1 they belong words and grammar, to the level 2 it belongs the creation of periods and their connections.

The differences between jazz, classical, pop languages belong to level 2: all of them use mostly the same syntactic material (the tonality with his rules) but they differ because of the style of language, the rhythms, timbres and aesthetic. Similarly to the literature, there are science fictions novels together with scientific essays, both written according to the same grammar rules.

This research will not treat these aspects.It specifically deals with the study of level 0 and level 1 of musical structure.

3.5 Towards 24 equidistant notes

We are now ending this analysis of music as language and we are approaching the creation of a method to transfer these properties into a quarter tones music.

I want to apologize with the reader if this section might have been a little bit to long and not on the actual focus of the research. Anyway it was the starting point of it; through this analysis of music from a semiotic point of view will let us better understand the axioms I have defined towards the creation of my method. All the reflections which have came up into the previous pages are not proper of the method I will through now. When I have developed it I did not reflected upon all the issues we have faced before. However they came up when I reflected upon the work and when I questioned it. Therefore I truly believe that they are

both interesting and relevant to the work, even if the core of this research is practical: it could be considered an harmony *traité*.

Starting from the analysis of music as a language, I worked on the creation of a twenty-four objects system; a language based on a twenty-four letters alphabet, twenty-four notes instead of twelve.

The division of the octave is no longer made up of twelve half steps, rather, the octave is divided in twenty-four quarter tones, doubling the letters available for the creation of musical functions.

Both the Dominium of the musical functions (L0) and the possible musical functions (L1) increase. However the three tonal areas (Tonic, Subdominant and Dominant) remain the same (but of course they organize a set of double number of notes). The functions we found into a twelve notes system are still valid into a twenty-four notes system and they can also be applied on lots of new starting points.

My research belongs to basic level of music, the syntax level.
It focuses only on notes/pitches (it will not be talked about other parameters such as timbers and duration of sounds in time, rhythm). ***My attempt here is to create a new system to organize pitches using twenty-four objects instead of twelve, without referring to any particular aesthetic.***

The history of music is the history of arbitrary choices in the definition of discrete values through frequencies (notes) and choice of language systems which organize them. My research is a further search for the creation of a system which uses notes. It differs from a lot of researches that has been made on micro-tonality since it tries to remain into the tonal music ambit.

We now approach the method in the following chapter.

Chapter 4

Methodology: generative process

While in the previous chapter the main focus was on semantics and philosophy issues regarding this research,; this chapter shows the mathematical and geometrical side of the work, approaching the proposed methodology.

In specific, after the long introduction previously faced, here I now present the method (which I call *geometric method*) I have used to organize a tonal system that uses an alphabet of twenty-four notes.

It came out after I have tested out other different methods. Therefore I start with an overlook to the various method I have approached before to adopt the proposed one. I do this because I think it might be interesting for the reader to meet the thinking process I have undertaken into the attempt to create a useful method. This might explain some choices later on adopted and it might also answer some "*what if you would have done like?...*" that the reader might argue here and there.

Chronologically, the first thing I did was to play quarter tones while I was testing out some tuning systems on the guitar. I was searching for some new sounds to enrich my instrument and while I was stretching my strings I stopped in the middle of the half step. As soon as I listened to those sounds, I immediately enjoyed their effects to my ears, and I started to think a way to organize these quarter tones in order to be able to think and to play them.

The research was therefore born in a *practical* context, it was born in middle of fingers and fret-board. The first thing I did was then to *play* with this new toy, experimenting different tunings and getting used to the sound of these new notes. It was certainly difficult in the beginning to get familiar with quarter tones, since I was not trained (in my musical studies) to hear them and consider them as tones. This was, and still is, one the most arduous peak to climb: to accept quarter tone as notes coming from a western music education.

Nevertheless I held on and I tried to ideate some music which would have used quarter tones. Then I immediately faced the definition of a method to compose with these notes, a method to organize them into a reasonable system. The first approach that came up after the practical one was *évidemment* the empirical one.

I took note of the various attempts I went through and I have analyzed them, in the hope to be able to identify some rules and consequently to define a method to conceive them into a useful frame.

The first biggest issue was the harmony. Concerning it, the first step was to understand how quarter tones sound like when played as a vertical object, when they are played together as chords.

After the first, purely empirical, experiments in which I just tried out lots of possible combinations of intervals and later on I reflected upon their quality according to my personal sensibility; I tried to approach the issue grouping quarter-tones according to functional harmony principle. Thus I started to work on diatonic scales made of seven notes, and I changed arbitrarily all the notes of the scale except the root.

This second approach was still empiric (let us alter these notes or those and see how it sounds): based on playing music and following the ear perception of results. I started from the scale models adopted in tonal music: minor melodic scale, major scale and minor harmonic scale. Then I shifted every note a quarter tone up or down randomly.

Even if there was a frame in which I conducted my work, I still evaluated the results just by listening them and giving them a personal evaluation. Yes, the adoption of a diatonic frame is already a great context definition, nevertheless I did not posed rules towards the choice of altering one note or the other.

This first method has produced a countless possibilities of mixtures and scales since I used all the major, the minor melodic, minor harmonic and pentatonic scales. This huge amount of results was, as the reader might has already argued, pretty hard to organize. However the peculiar characteristic of this method was the close connection to scale models we all are used into the western tradition. Therefore it was easier to conceive quarter tones into this familiar frame but (there is always some "but") quarter tones appeared only as a weird distortion of the tempered tones. Quarter tones still did not have the dignity of proper notes with their own unique harmonic and melodic functions: they looked as *ornement*.

I have thus to recognize the merit to this method to have got me used to quarter tones into a more familiar context. On the other hand this method carried two great deficiency: it was still too empirical and it did not grant a real *note function* to quarter tones.

Furthermore, regarding the creation of chords, this method did not contribute so much into the clarification of my ideas upon that subject. I found some clusters and chords inscribed into a scalar context but no more.

By the way, this empirical work has showed me a lot of interesting results; especially I became more aware of the existence of new scales with quarter-tones, which are not so far

from some extra European music, especially the Arabic one in Maqām. [1]

Again, on the other hand, this last approach I tested out was not organizing sounds into a rational system rather it was based on the creation of scales trusted on chance and stuck into an empiric system.

These empirical approaches have produced in some case good results, but surely they could not be the basis to found a general model to approach the problem. ***I then felt the need to adopt a structured and more formal approach, defining a proper methodology.***

I did not feel comfortable without a method on which I could have relied on. It was not enough to rely on instinct and trust my perceptions. Since I was definitely not used to quarter tones I needed some rules to rely on, a system that could have guided me into the composition in order to have some coherent music to evaluate later on. Then yes, it would have felt okay to express some personal judgements on the music, but I needed some more structured skeleton to rely on before.

Therefore I moved on with another method. In the following system I approached, I worked on tritones trying to discover new cadences coming out from the resolution of quarter-tones intervals.

This method focuses on chords progressions and proposes resolution movements which involves quarter tones steps. I propose here a brief example of its functioning. Playing a triton, for example F/B, you can resolve it whether traditionally to C/E going up or down a halftone, or you can resolve it to C-/E+, going respectively up and down only a quarter tone.

Both the thirds C/E and C-/E+ resolve the tension of the triton, therefore they can be both considered valid for the construction of an harmonic movement. In this way, it is possible to obtain new cadences between tuned chords with their own internal tuned structure, but put into a distance made up of quarter-tones. This method looked very inspiring and furthermore it can be considered less empiric, more structured then the previous ones. As the reader can observe, this method starts from the alteration of a constitutive principle of the music language rules. It takes moves from the concepts of consonance and dissonance and their movements regulation. It also does not introduce any new vertical intervals, it only introduces new intervals in the horizontal developing of the music. This helps the listener to not get dizzy facing some *quarter tone cluster*. However it introduces new harmonies giving to the listener the opportunity to get more and more used to quarter tones sonority.

Nevertheless, during my research I have soon found out that composers as Alois Hába and Ivan Wyschnegradsky have already undertaken this path before in their works (during the 30's). Also nowadays an Italian colleague which I am collaborating with, Stefano Risso, is continuing these studies.

As said previously, this method is not empiric, thus it represent a further step towards

[1] David Muallem. *The Maqām book.* Or-Tav Music publications, 2010.

the creation of a proper methodology.

However it did not allow me to identify some scales to relate to the chord progressions. Therefore I felt that it was interesting but yet not complete. I needed a method that could have grant me the opportunity to compose and improvise with quarter tones, thereby I needed scales generating tonal functions and therefore harmonies.

Thus I moved on from this method trying to discover a way to conceive a more complete musical overview on the topic of quarter tones. In other words, at this point in my research I clearly felt the need to organize these sounds through a different system, that could have been complete and coherent with the musical result.

I approached a method that made use of some external features to determine a choice of notes, both to generate scales, both harmonies. Therefore I started considering all the twenty-four notes and then I tried to organize them through geometries.

Claiming Slominsky, **I decided to construct a method based on geometrical symmetries.**

Before to deepen into the explanation of the method proposed, I here insert a more precise formalization of the concepts of tonal function, I propose a more precise definition of tonal scale as modular additive operation.

4.1 Tonal functions as geometrical figures

This section uses a specific technical language.
Therefore the reader who is not practical with math can jump over it and go on to the next section.

However I felt the need to state here a more precise definition of the concepts mentioned previously in chapter 3.

I want here to thanks again Valter Degani who has helped me with this language, not really familiar to me.

Let us consider a mathematical formalization of the concepts and the ideas of musical scale and tonality, that have been previously exposed (see chapter 3).

First of all, we define the set L of the elements (the notes) in an octave, as a finite and ordered set of n elements: $\mathrm{L} = \{a_1 \dots a_n\}$, with the relation order $\prec$. [2]

[2] given a set A, an order relation $\prec$ is a binary relation such that, $\forall x, y \in A$:

- $x \prec x$
- $x \prec y$ AND $y \prec z \Rightarrow x \prec z$
- $x \prec y$ AND $y \prec x \Rightarrow x = y$

The order relation that rules the elements is the one implicitly defined by elements numbering: $a_i \prec a_j \Leftrightarrow i <= j$.
Given two notes x and y, if x $\prec$ y, then we say that x is minor, or it came first of y (or that y is major, or follows x).

The number n, the cardinality of the set L, is also called *the size of an octave.*

This order naturally implies a way to correlate two generic notes in an octave, which is formalized by the two functions [3]:

- RelativeDistance(a_i,a_j)=|i-j| .
- Distance(a_i,a_j) = j-i = -Distance(a_j,a_i) .

It is said that the element a_{i+k} has a fixed distance of k steps from the previous element a_i (or the element a_i has a distance of -k steps from the element a_{i+k}) and therefore their relative distance is k. We also say that a_{i+k} is at k steps further from a_i (or that a_i is k steps less from a_{i+k}).

Given an octave L^k= $\{a_1^k \dots a_n^k\}$, it is always defined its major octave $L^{k+1} = \{a_1^{k+1} \dots a_n^{k+1}\}$ and its minor octave $L^{k-1} = \{a_1^{k-1} \dots a_n^{k-1}\}$; formally, the octaves are infinite, and enumerated with a relative integer, defining a natural order on the octaves. The set of all notes between all the L^k octaves is named N.

The order on the notes into an octave and between octaves, implies an order on set N of all the notes between generic octaves:
$$\dots \underbrace{a_1^k \prec \dots \prec a_n^k}_{\text{octave k}} \prec \underbrace{a^{k+1} \prec \dots \prec a_n^{k+1}}_{\text{octave k+1}} \dots$$

We have used the notation x^{+j} (x^{-j} to indicate the same note x but on the octave that is more (less) j octaves with respect to the octave to which x belongs.

Each note of a given octave is minor that all notes on major octave, and the last note of an octave is at distance -1 from the first note of the successive octave.

So both the concept of RelativeDistance and Distance can be extended from notes into an octave L to all the notes between octaves N.

The relative distance between two notes in N is also call a *music interval.*

[3] the distance between two notes is a concept that contains more information respect to the concept of relative distance. Indeed the function Distance(x,y) enable to deduce if the element x precedes or follows another element y; whilst the function RelativeDistance does not.
As an example:
RelativeDistance(a_5,a_3)= |3-5|=2; Distance(a_3,a_5)=2; Distance(a_5,a_3)= (3-5)= -2.
So, by knowing that Distance(a,b)=3 , it is possible to deduce that a came first of b; that is if a= a_i then b= a_{i+3} .

The step defines the *unit of measurement* for the distance between two generic consecutive elements in N.

In this model it is not relevant to consider a single note and its characteristics; it is relevant only the relation between notes formalized by the concept of (relative) distance. [4] The concept of relative distance defines the fundamental relation within two notes of N: we say that two notes are in relation of 5 to denote that their relative distance is 5.

Starting from this formal context we can immediately define an operation to obtain a note starting from another one: the unary operation +, called add. It generates the note at the distance of 1 step from the given one which its successive note based on the order defined on N: $+(a_i) = a_{i+1}$. [5]
By generalization, iterating k times this operation, we obtain, from a note a_i, the note a_{i+k}: $+^k(a_i) = \underbrace{+(+(+\ldots+(a_i)\ldots))}_{\text{k times}} = a_{i+k}$.
This operation is defined on all the notes N in all the octaves. It is necessary to constitute a definition of this operation which can be valid also considering only the *set of the elements* enclosure into a single octave. Furthermore, when we apply the operation on the last element a_n of a specific octave L^k, we will obtain the starting note of the successive octave L^{k+1} [6]

For each couple of notes in N, the relative distance is given by the number of iteration of the add operation that must be applied on the minor note in order to obtain the major one.

Starting from one generic note in N, it is possible to add an arbitrary number of steps in order to obtain a new note (in the same octave or into another, further, octave).
As example, if the number of elements in an octave is n=4, then, starting from a_2 of a given octave:
a_2^k; $+(a_2^k) = a_3^k$; $+(a_3^k) = a_4^k$; $+(a_4^k) = a_1^{k+1}$; $+(a_1^{k+1}) = a_2^{k+1}$; ...

To simplify the formulae, it is defined one single new additive binary function $+:\mathrm{N}\times\mathbb{N} \mapsto \mathrm{N}$ as: $+(a_i,\mathrm{k}) = +^k(a_i) = a_{i+k}$. This operation adds k unitary steps to an element to obtain another element.

It is therefore obviously immediately introduced on N the symmetric unary operation - *of decrease of one step*, simply called subtract, as: $-(a_i) = a_{i-1}$ e $-^k(a_i) = a_{i-k}$.
Furthermore with this operation it is possible, starting from one generic note in N, to subtract an arbitrary number of steps in order to obtain a new note (in the same or the another further octave).
For example, if n=4, then starting from a_2 of a given octave:

[4] if we consider it from a physical perspective instead of an only formal one, the concept of distance between two notes can be explained as the difference between the pitch values of the two notes; but this consideration is completely irrelevant for the way of reasoning that is here formalized.

[5] the operation is called add because add 1 step to a note to obtain the follow note.

[6] Recalling the physical interpretation of the distance between two notes as the difference about their pitches, regarding the music theory ambit, we say that the note a_{i+n} is 1 octave major than the note a_i.

a_2^k; -(a_2^k)= a_1^k; -(a_1^k)= a_4^{k-1}; -(a_4^{k-1})= a_3^{kj-1}; -(a_3^{k-1})= a_2^{k-1}; ...

This formalization expresses clearly how the process which is used to generate notes in music is the same one used to generate the Natural numbers, as described by Peano. We might then argue that the structure of the music syntax is based on the logic structure of arithmetics, which is the fundament of the entire mathematic. [7]

It is efficient to represent the properties of N and its internal organization as an ordered sequence of L^k octave sets, through a geometric construction. Let us define a circumference; consequently we uniformly distribute all the L elements of a generic octave on its boundary.

[7] In the *quadrivium* postulated by Marziano Capella before the first millenium AD, music has been gathered together with other disciplines such as math and geometry. We all might argue that there is therefore a deep connection between music and math and furthermore that there is some common ground which relates these two disciplines apparently so far one from each other.

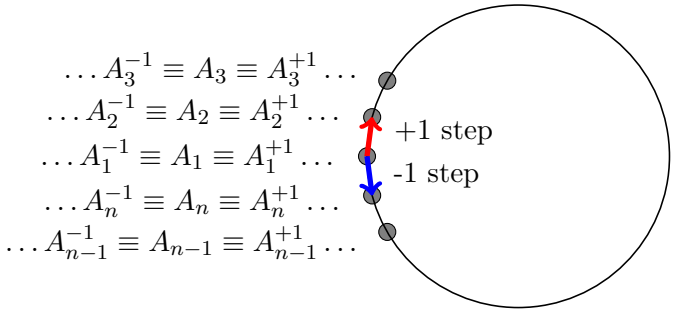


Figure 4.1: L circumference of notes

The effective position on the circumference boundary to which a specific note is related, is not relevant. On the other hand it is relevant the fact that all the notes are on the boundary at a relative equal distance (in order to represent the concept of single step): a generic note has a distance from the previous one and the following one which is constant and therefore always the same.

A specific direction on the circumference defines the order relation in L and N (that is valid for all the notes on all the octaves) regarding the notes (the succession of notes which is in accordance with the defined direction identifies the elements with an increasing order, from a note to its successor). The operation defined as "*add, or subtract, a unit step operation*", is geometrically equivalent to go in the circumference from one note to its successor (or predecessor).

Having in mind this geometric representation, the operation to add (or subtract) a step is obvious; walking on the circumference bounder, after having reached all the notes in the octave, we again reach the same notes but on the increase octave, indefinitely. This is due to the fact that each point in the circumference represent a note x *but in all the octaves.*

The relative distance between two notes *when it is minor or equal to the size of the octave* (i.e. is minor or equal to n), can be represented by the relative length of the segment that joins the elements, that can be deduced by the number of steps it has to be taken from the minor to the major note (in other words, if we want to represent the distance, then we have to use an arrow).
This circumference is called the L circumference, or simply the circumference of notes.

4.1.1 The Tonal function.

After the definition of the context in which notes are disposed, we now approach the definition of the the concept of *tonal function.*

A tonal function is a function such that for each element of N, it relates an ordered sequence of elements of N, which must be placed to a defined relative distance (musical interval) one to each other.
This set of notes is called scale of the tonality, and the notes are called notes of the scale.

A tonal function is possible to define a scale for each notes in N.

This idea can be formalized using the concept of function of superior order k.
A tonal function is a function from N $\mapsto N^{k+1}$ where [8] the set of notes to be defined are obtained using an ordered sequence, Tonality = $[n_1 \dots n_k, n_{k+1}]$ of numbers in the set $\{1 \dots n\}$, (where n is the number of element in L), such that: $n_1 + \dots n_{k+1}$=n.
The elements of the ordered sequence are call *intervals*, and the ordered sequence is called *ordered intervals.*

This sequence defines an *iterative additive mechanism* that, starting from the initial note x (the independent variable of the tonal function), identifies all the k+1 notes that define the scales of the tonality, following the rule:
x, $x + n_1, x + n_1 + n_2, \dots, x + n_1 + \dots + n_{k+1}$. [9]

Please, note that in the definition of tonal function we can talk of Tonality as an ordered set, not a simple set. This is a necessary consideration since we want to reflect upon an ordered set of elements. Thereby, two tonal functions defined by two ordered sequence which differ only by the order of the element and not by the values, are different.

The tonal function characterizes scales that all have the same number of notes with a relative distances which is defined by the ordered sequence of the tonal function.

A tonal function [10]identifies a set of notes with same relative distance (it transcends the specific notes) but it consider only the relative distance, and its order.

As example, let us consider, starting from a set L of 12 notes, {C,C♯,D,D♯,E,F,F♯,G,G♯,A,A♯,B}, the tonal function $\alpha : L0 \mapsto L0^k$, [1,3,5,3]. This tonal function identifies:

on the note A, the scale: $\alpha(A)$ = A, A♯ (A+1), C♯ (A+1+3), F♯ (A+1+3+5);

on C, the scale: $\alpha(C)$ = C, C♯ (A+1) , E (C+1+3), A (C+1+3+5) ;

If we define a new tonal function β, [1,3,3,5] , that has the same set of value of α, but in a different order, the scale obtained on A, is:
$\beta(A)$ = A, A♯ (A+1), C♯ (A+1+3), E (A+1+3+3); that is different from $\alpha(A)$.

[8] in mathematic, given a set A, the set A^n is a set which elements are n-ple $<a_1 \dots a_n>$ of elements of A. An n-ple is a set that consider also the order of its element; so as example <1,2,3,4> is a different object from <2,1,3,4>.

[9] By definition, $n_1 + \dots n_{k+1}$=n (or equivalently n_{k+1} = Distance(x+n_k, x)).
So $x + n_1 + \dots n_{k+1} = x^{+1}$; the last additive step lead back to the start note in the superior octave. This property is called *modularity property of the additive sequence of a tonal function.*

[10] A tonal function is completely defined by its ordered sequence Tonality. By sake of simplicity, we can identify the two concept and write a sentence such as: "considering the tonal function $[x_1, x_2, x_3, x_4]$..."
Another mode used in the text in order to represented a tonal function is: $[x_1 + x_2 + x_3 + x_4]$. This has been used in order to emphasize that tonal function is an additional sequence to generate notes. Thus, it will be possible to write sentences such as: **"Considering the tonal function [1,3,3,5]", or "Considering the tonal function [1+3+3+5]"...**

It is very important to highlight that the concept of tonal function is completely defined into the ambit of a single octave. The idea is that what is relevant is the relative distance (the music interval) between notes and not the effective notes that are involved.

A scale of a tonal function and the *tonality function* can be obviously represented through a geometric construction, using the previously described concept of circumference of notes.

A scale of a tonal function is representable as a closed geometric figure inscribed into the circumference of notes, in which each vertex of the figure represent one note of the scale, and each side of the figure defines the relative distance between the related notes. [11]
A scale from a tonal function is characterize by the fact that all its notes have a relative distance at most of the size of one octave. Furthermore the figure is surely closed, because the additive sequence is modular.
The *relative length of a side is defined as the number of steps necessary to reach one vertex of the side starting from one other*; thus the relative distance between two notes, involved by a side, constitutes the relative length of the side.

For example, the scale $\alpha(\mathrm{C})$ and $\alpha(A\sharp)$ of the previously defined tonal function α, can be geometrically represented as:

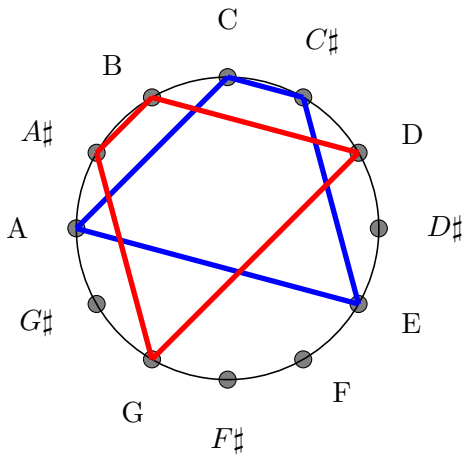


Figure 4.2: Geometric representation of scales

As the reader might have understood, the geometric representation of two different scales of the same tonal function implies simply a *translation* of the same figure into the circumference of notes.
So, a tonal function can be geometrically represented by the representation of one of its scale, considering that all other scale are obtainable translating the figure on all the other note points of the circumference; and that a geometric representation of a scale also describes

[11] When we talk about *scale of a tonal function* we mean the application of the tonal function on a specific note; in other words we mean a specific scale. For example if we consider the tonal function *major scale*, then one of its scale could be the *C major scale* as well as the *A major scale* and so on.

the relative distances of the additive sequence that completely characterize the tonal function. The scale used to describe the tonal function is called the *reference scale of the tonal function*; and this representation of a tonal function is called *implicit geometric representation of a tonal function by the reference scale.*

More formally, we can consider the circumference of notes as a representation uniquely of their relative distance, not of a set of specific notes.
Therefore, now a point defines only a generic note, that is at distance +1 step from the previous one, and -1 step from the successive one. Each point is labeled by its relative distance from the *initial one, that is labeled as 0, that is equivalent to the number of notes in L* (As usual, the effective position of the initial point 0 is not relevant).

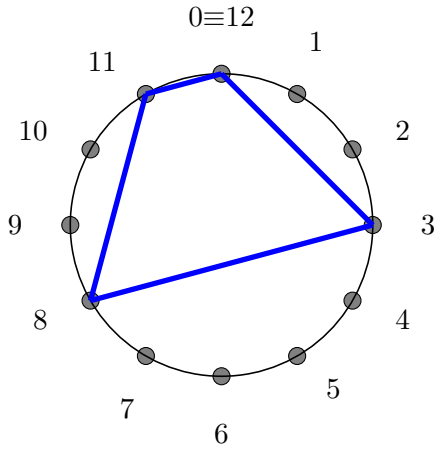


Figure 4.3: Geometric representation of a tonal function

Then, a tonal function is representable as a closed geometric figure inscribed into the notes circumference, when its ordered additive sequence can be obtained considering the relative length of each side. To compute the additive sequence of the tonal function, we can simply start from the initial point, and compute each side relative length in sequence as the difference between the two labels of the involved points. In the example reported in the figure we have considered the tonal function: [3-0, 8-3, 11-8, 12-11] = [3, 5, 3, 1] .

Summarizing, given the circumference of notes, each close figure inscribed into it defines a specific tonal function. The additive sequence of the tonal function can be obtained simple computing the relative length of each side of the figure. In the case that we use an implicit geometric representation of a tonal function by a reference scale, the computation of the relative length of a side can be obviously obtained considering the relative distance between the involved notes of the side, as defined by the order in L.

From all this, it immediately derives that each set of at least three points of the circumference of notes (i.e. each set of notes) defines a specific tonal function: exactly the one that is related to the closed geometric figure which has as its vertexes these points. The set of vertexes are said to be the *generating vertexes of the tonal function.*

To identify a tonal function and to study its characteristics, it is a problem that can be approached considering the geometrical closed figures inscribed into the circumference of notes and some of their geometric properties.

This is one of the principle ideas that has been used in this work: ***to define a tonal function starting from some geometrical properties of its related geometric figure.***

4.1.2 The Tonal areas.

Let us consider a tonal function α and its ordered sequence, Tonality $= \{n_1, \dots n_k, n_{k+1}\}$.

We can now define 3 subsets of Tonality: the Tonal Areas. These three sets are named Tonic, Dominant, Subdominant. These sets must cover all the Tonality set, but they do not necessary define a partition [12]:

- Each element of Tonality must belong to at least one set of tonal area sets.
- A specific element of Tonality can belong to more than one tonal area sets.

Given a scale α(x), we say that the note x+$n_1 + \dots + n_i$ of the scale belongs to a certain tonal area set T, if $n_i \in T$.

Within the tonal areas it is defined a specific order relation: Subdominant$\prec$Dominant$\prec$Tonic .

This order regarding the sets implies a partial order relation [13] between the notes of a tonal function scale: $a_i \prec a_j \Leftrightarrow a_i \in A, a_j \in B$ and A $\prec$ B, where A,B are 2 of the 3 tonal area sets.

The order between the notes is partial because it does not define any order between the notes that belong to the same tonal area.

4.1.3 Musical movements, cadences.

In the music theory, the tonal areas are used to reason regarding movements among pitches, both into an horizontal and vertical way.
The horizontal movement is related to the order relation on the tonal areas.
Verticality it is defined by the tonal areas itself.

It is important to highlight that ***if a tonal function is completely symmetric (all the element of the additive sequence are equals, i.e. all the notes of the related scales have the same relative distance, i.e. the related geometrical figure is a completely symmetric figure), then the tonal area sets are meaningless: each***

[12] A partition of a set A is the division of the set in a finite number of subsets, when each element belongs to one and only one subset. As example from {a,b,c,d}: {a,d} {b,c,}.

[13] in mathematics, regarding the assertion that the order into a set is partial, we can assume it when not all the elements have a valid order relation.

note belongs to all the area tonal sets.

We now move on from the mathematical formalization into the definition of the generative section of the method.

Before to go further I want to place here a little table which is meant to clarify the quarter tone notation used in the text.

4.2 Quarter tones notation while used in text.

It is here set a small legenda regarding quarter tone notation into text writings.
This small section is meant to clarify the notation that will be used to write quarter tones further in the text.

Note value	**Textual represention**
C -$\frac{3}{4}$	C♭-
C -$\frac{2}{4}$	C♭
C -$\frac{1}{4}$	C♭+ , C𝄳 , C-
C	C
C+$\frac{1}{4}$	C♯- , C𝄲 , C+
C+$\frac{2}{4}$	C♯
C+$\frac{3}{4}$	C♯+

Table 4.1: Textual notation for quarter tones.

Please, note that here it has been introduced a specific notation to label the quarter tone accidents: 𝄲 and 𝄳.
The first symbol (𝄲) means that the relative note is increment of a quarter tone; the second symbol (𝄳) means that the relative note is decremented of a quarter tone.
So, as example C𝄲 is equal to C $+\frac{1}{4}$ and D𝄳 is equal to D -$\frac{1}{4}$.

4.3 General considerations

Before to start the treatise of the method I want to make clear the way I have proceeded from the beginning to the end of it.

The method is articulated in two general steps (as said in the introduction, see chapter 2) which can in turn be divided into further steps. Firstly I have defined some processes to generate scales according to geometric principles (which is this section, the generative section of the method); later on I have selected among the result, later on I have selected among

the results. As it will be clear later in chapter 5, this second section of the method can be divided into two sections as well.

Firstly I have adopted again a geometric principle to divide the results obtained here and obtain more various results, then I have applied musical processes (along with semantic reflections) to re-organize the material into a different classification.

Therefore the scales obtained by the method will belong to multiple classifications, a purely geometrical one and a more musical one which in turn will operate a partial generative action.

Made this premise, I now go on with the treatise of the first section of the method.

Towards the construction of this method, ***The main goal is to obtain tonal functions, into an alphabet of twenty-four notes, in which is possible to locate the tonal areas S D T.*** [14]

This is because (as said previously in the text) this research wants to explore *tonal related music using quarter tone.* It would not be necessary to go deep into the formalization of tonal scales if we would not aim to reproduce tonal syntax into a twenty-four objects domain. Therefore the method here proposed is an attempt to *generate tonal functions using notes which has never been used before into this context.*

In the beginning, I started to study some of Messiaen [15] and Slominsky, whose talk about the possibility to group twelve half notes following rules of symmetry. In his *Thesaurus of scales and melodic patterns*, [16] Slominsky begins his theory adopting a division of the octave according symmetric portions, going on with the same schema through "infrapolation" and "ultrapolation" to create scales. Therefore to him, a tonal function is intended as an application of some symmetry properties on the relative distances, defining an order sequence into an octave.
Starting from this concept, it came up to my mind to adopt an ***abstract and formal method*** *to organize 24 quarter notes.*

As it might be clear, this method has a deep connection with Math and Geometry, and therefore it is possible to conceive it because as first step I have preliminary analyzed abstractly all the concepts related to a tonal construction process: the tonal function, the tonal areas and the musical movements. The formalization of tonal music into a mathematical frame led me to go deeper into geometric construction of scales initiated by Slominsky.

From the previous mathematical analysis (See 4.1) it is clear that a tonal function organizes music intervals and that it can be represented as a plane geometric figure. The method is thus based on the manipulation of these two concepts: it proposes specific geometric properties on the figures and it relates numeric properties on the intervals this way founded. More generally: I relied to some geometric principles in the belief that they could have led to a

[14] As said before, vertical functions are not treated here; they could be deepen in further works. See 3.4.1
[15] Olivier Messiaen. *Tecnica del mio linguaggio musicale.* Paris: Alphonce Leduc, 1999.
[16] Nicolas Slominsky. *Thesaurus of scales and melodic patterns.*

valid organization system.

Generally this method is linked to Math and Geometry in a double way:

firstly it starts from the consideration of scales adopting formalization items coming from the consideration of scales adopting formalization items coming from these two disciplines;

on the other hand it explores geometric properties of basic figures on which it relies in order to obtain valuable organizations of notes.

Therefore this method poses a great faith into the uncontrovertible properties of basic geometries, in the hope that they could lead to some interesting musical result.

Furthermore I do not believe that music is ruled by geometries, I basically use this comparison as inspiration, as limitation to overtake the musical limits I would have faced into the conception of a twenty-four notes music.

On the other hand, into the qualitative section of the method, I utilize music criterions to valuate the scale geometrically obtained. There I adopt some semiotics reflections along with musical theory principles to drive back the result here got towards a useful musical result. The reader has to keep in mind this double feature of the method.

As previously explained, the geometrical representation of a tonal function is based on the concept of circumference of notes, where it is arbitrary defined a specific (and conceptually not relevant) position for each of the 24 quarter tone notes. The circumference of 24 notes that will be used later is explained in the following figure:

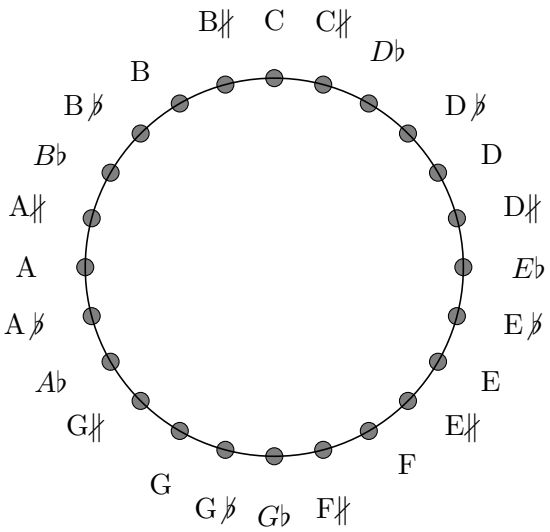


Figure 4.4: Circumference of 24 notes

Please, note that here it has been used the specific notation previously explained to label the quarter tone accidents: 𝄲 and 𝄳.

Finally, please note that the direction on the circle congruent with the increasing quarter tone order is from left to right.

An important consideration that has to be stated now is that it is fundamental to **verify if the scales selected by the proposed method to have a sufficient number of elements in order to be considered valid: generated scales must be constituted by at least five elements).** This is because we consider a four notes scale same as a chord, we can not accept organization systems this little otherwise it would be impossible to recognize a property to the elements (they would be too little).

This has been taken as a structural premise towards the creation of the system. After this axioms, the method is now explained in details.

As we see on the following pages, they have been identified two general concepts which are fundamental in order to be able to produce tonal functions and scales. They can be considered the core decisions of the generative process.

4.4 Absolute similarity principle

As previously explained, a tonal function is defined by its series of ordered relative distances (intervals). Starting from the theory of *interval areas* proposed by Franco D'Andrea. [17], we define the concept of *absolute similarity principle*, an important relation principle regarding

[17] Franco D'Andrea and L. Ranghino. *Aree intervallari.* Volonté & Co, 2011.

tonal functions.

D'Andrea is proposing an organization of tones ruled by the reiteration of a couple of intervals. He proposes as similar and then possible to be gathered, all the series of notes which respect the boundary of movement stated in the *interval area.*

For example let us consider the following area: right fourth + major second.

Now they will be possible all the combinations of notes which make follow a fourth or a tone to an arbitrary tone. This means that we will consider similar fragments possibly belonging to different tonal ambit and furthermore we will consider similar different tonal functions because they all share (even if in a different order) the same values of relative distances, the same intervals.

Therefore, elaborating the concept of *interval area*, in my method two different tonal functions are called *absolute similar* if they use the same distances and the same number of elements eventually with a different order.

For example the tonal functions [2+5+2+5] and [2+2+5+5] are absolute similar, because they share both number of elements (they are both a four elements tonal function) and values (2 and 5). They generate different scales (from C we would have: D♭; E𝄫; F𝄫; G concerning [2+5+2+5] and D♭; D; E♮; F♯ concerning [2+2+5+5]) but they will be gathered and considered *absolute similar* according to the principle just stated.

If we borrow this principle from the tonal functions to their related geometrically representing polygons, we will consider *absolute similar* closed figures with the same number of sides with the same length.
For example, considering the octave set L with the Domain of 12 elements, the two quadrilateral figures [4,5,2,1] and [2,4,5,1] [18] will be absolute similar:

In conclusion, we consider two plane figures composed by the same number of sides of the same relative length inscribed into the circle of notes as absolute similar; independently by other geometrical characteristics such as the area or the angles.

From a musical point of view, the absolute similarity principle proposes to consider equivalent scales that organize the same set of music intervals in different order (close to the concept of *Area intervallare* expressed by Franco D'Andrea). This means that there will be different scales which share the same constructive intervals and therefore somehow the same *color, flavour, structural skeleton.*

[18] Of course here we are considering polygons which describe tonal functions. Thereby figures are described by their relative tonal function values. The tonal functions, geometrically represented, in order to draw to cumulative absolute function, must share the same independent variable in order to be put together starting from the same note. Since tonal functions gathered by the *absolute similarity principle* share the same values, the order is strictly relevant. Thereby when we decide to draw f(C) instead of a generic f(x) we must do the same for all the other absolute similar functions.

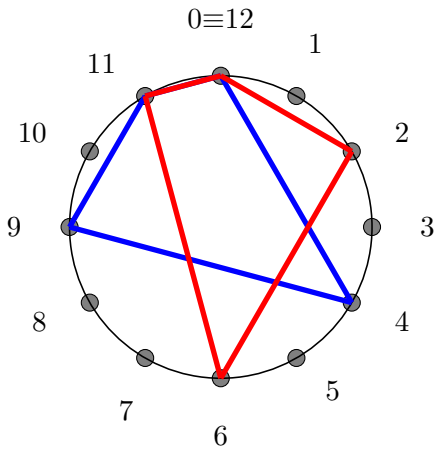


Figure 4.5: Absolute similar figures.

We can observe that the *absolute similarity principle defines a further abstraction which gives another property to tonal functions: different tonal functions now share a common geometric property which connect them.* This mean that there will be a context where different tonal functions will belong; *a context which defines itself a cumulative tonal function.*

More formally: let us consider a set of different tonal functions; we define *union of the set of tonal functions* the tonal function that has as vertexes the union of all the set of vertexes of the cumulative tonal functions this way merged. The cumulative tonal function is thus the tonal function constituted by the sum of all the absolute similar tonal functions. The construction of the cumulative function implies the same independent variable for all the tonal functions gathered by the absolute similarity principle. In other words: *these functions has to start from the same note.*

Of course, in the case that the merged tonal functions have been represented as an implicit geometric representation from a reference scale, then *the scale must be the same for all the functions*; put it in another way, it is the value of the independent variable that defines the scale which must be always the same.

We call *closure of a tonal function by absolute similarity principle*, or simpler, absolute tonal function, the tonal function obtained by the union of all the tonal functions that are absolute similar to each others (cumulative tonal function). The other way around: the set of tonal functions that are merged by the absolute similarity principle are called the *constituent figures of the absolute tonal function.*
Again, the constituent figures of an absolute tonal function share the same elements: the only difference regards their order; consequently, the number of aggregated tonal functions is the number of possible permutations into the set of involved relative distances.

Of course, each couple of tonal functions that are absolute similar share the same cumulative tonal function. By the way if we define a cumulative function related to a tonal

function we can of course do the same for all the others absolute similar functions. Thus there is a relation between a cumulative function and several tonal functions.

For example, let us consider the octave set L constituted by 12 elements, and let us calculate the absolute function of the set {2,3,7}. This is a valid request, since [2,3,7] is a tonal function (2+3+7=12 and 2,3,7 $\in \{1, \ldots, 12\}$).
First, the set of tonal functions that are absolute similar are these 6:
t_1= [2,3,7]
t_2= [2,7,3]
t_3= [3,7,2]
t_4= [3,2,7]
t_5= [7,2,3]
t_6= [7,3,2]

To describe the union of this set of tonal function, we use the implicit geometric representation of the tonal functions by the reference scale defined on the note C. All the merged tonal functions are represented with blue figures. The obtained absolute tonal function is draw in red.

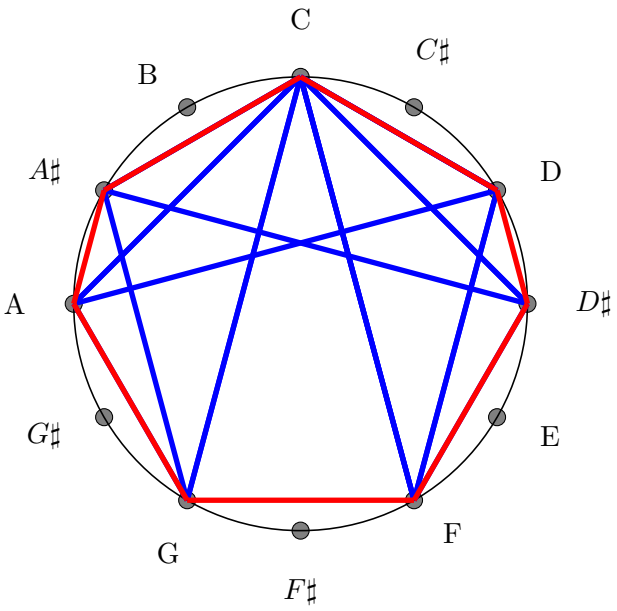


Figure 4.6: Construction of an absolute tonal function.

Finally the absolute tonal function from the set {2,3,7} is thus the tonal function {2,1,2,2,2,1,2}.

Another important statement: **the method considers as relevant only the absolute tonal functions, i.e. tonal functions obtained by closure by absolute similarity.** It gathers together, in order to generate scales, all the tonal functions which are related by the Absolute similarity principle.

The absolute similarity principle is then the first concept that characterizes the proposed generative method to produce tonal functions.

4.5 Symmetries properties approach

As it emerge from last section, the absolute similarity principle has a central role in the proposed method. The central idea is to define some specific geometric properties to select figures on which to apply the cumulative process by absolute similarity property, and then to obtain an absolute tonal function of interest.
Consequently, it is needed that *the selected properties are invariant in functions which respect the Absolute similarity property.* In other words: two different figures that are similar by the absolute similarity principle must both satisfy the selected properties.
The absolute similarity principle regards only the relative distances into the tonal functions ordered sequence; thus, it is relevant to consider properties that remain invariant from the set of absolute similar tonal functions and the cumulative absolute function. Thereby we will only consider the number of elements and their value, or regarding the correlated polygons, the number of layers and their length.

From another point of view, it has been decided to choose the term **symmetry** meaning *the principle geometric property used to organize tonal functions.* This decision is based on the consideration that symmetry concept is a very generic principle and therefore it is applicable in various science fields such as in arts and nature. [19] Therefore the geometric properties that we propose to consider are the number of sides and the translation of the symmetry concept related to the length of the figure sides; given a figure with a fixed number of sides, we can simply consider the number of sets of sides with the same length; and thus consider symmetric a figure in which this number is one or two.

In conclusion, we introduce the concept of *perfect symmetry* and of *partial symmetry.* A figure *of a specific number of sides* is:

perfect symmetric if *all* its sides are of equal length. As example equilateral triangles, or squares, or pentagons. This figures are then called *equilateral figures.*

partial symmetric if *the number of sets of sides of the same length* is two. As example, rectangles, diamonds and isosceles triangles.

The proposed method consider only the figures that are perfect or partial symmetric in order to generate absolute tonal functions.

This approach set this method as an extension of the original approach proposed by Slominsky and and Messiaen [20]: when their method was only related to what we called perfect symmetries; here we extend the analysis to the partial symmetries.

This is the second concept that characterizes the proposed generative method to produce tonal functions: the adoption of partial symmetries relation.

Explained these two former concepts, I now exemplify its working process.

4.5.1 Perfect symmetries

The perfect symmetric objects are represented by the equilateral figures. Obviously, perfect symmetry creates always a limited number of elements that are *exactly equidistant.* As previously discussed (see section 4.1) the equidistance (which we can name also the equivalence of the values), causes automatically the decay of the tonal areas. Put in another way, if all the notes are equidistant, it will not have any sense to start from a note or another, it will not exist an order and the tonal areas with their grades. [21] *However these functions carry special characteristics and they are very important in music thus it is not possible to miss them.* Usually perfect symmetric scales can describe multiple dominant chords, according to the diminished chords theory of Barry Harris.

[19] One of many interesting books that propose and analyze in depth this concept is:
Michael S. Schneider. *A beginner's guide to constructing the universe.* New York: Harper, 1995.

[20] Of course my method deal with 24 notes instead of 12, nevertheless I want to highlight that my method is an extension of the Slominsky and Messiaen method *also regarding the symmetry property considered.*

[21] Moreover, please note that in this case the Absolute similarity principle is meaningless, because there are no tonal functions which are different considering the order of their intervals.

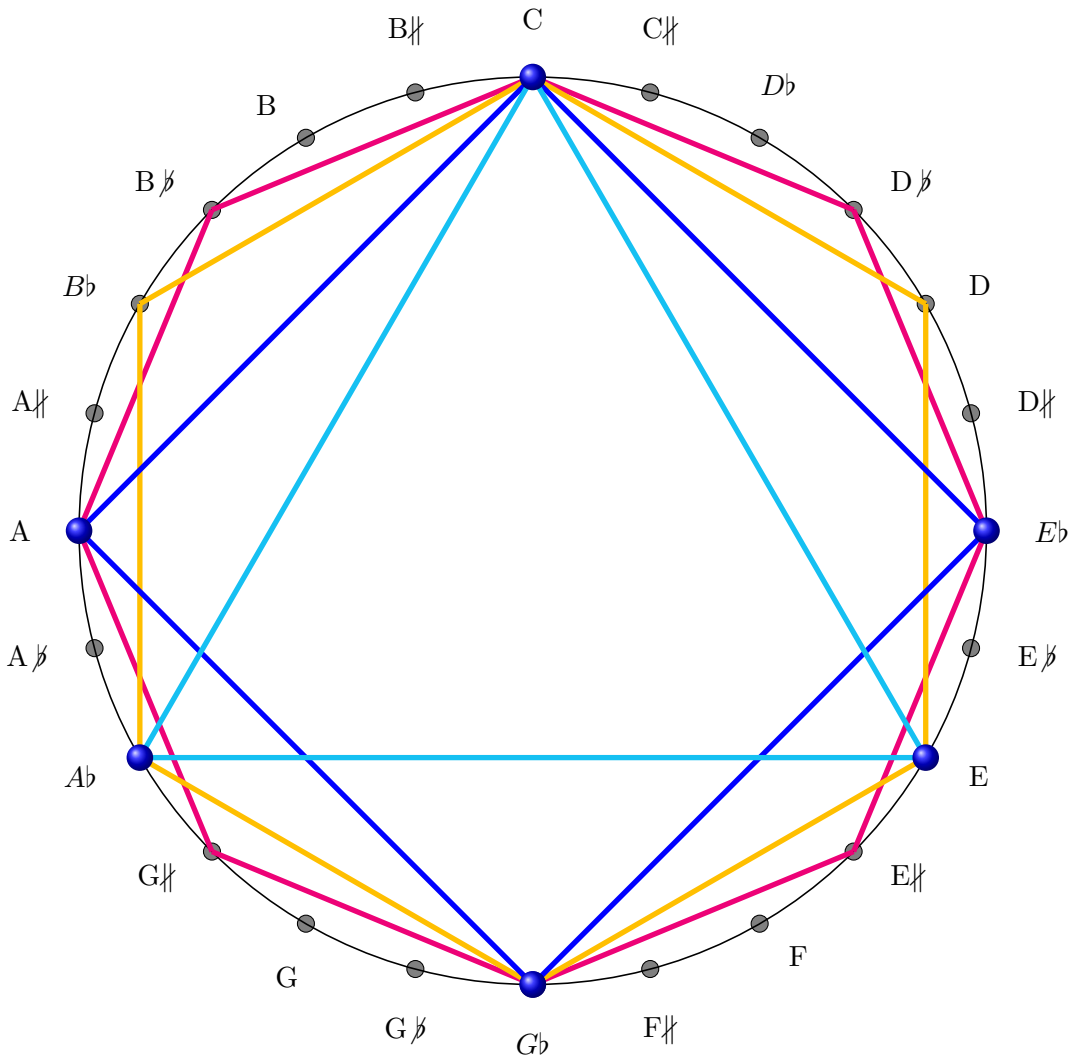


Figure 4.7: Equilater figures

The equilateral figures that could be inscribed in the circle of 24 quarter notes are defined by the divisors of 24: 3,4,6,8. Please note that while divisors 3,4 and 6 are also divisor of 12, so they can be found also into a twelve tone system. On the other hand, the divisor 8 exists only into a twenty-four notes system. So in our context of twenty-four notes system we have found a *new symmetric scale, the* ***octagon scale.*** When we convert these results into geometrical representations we got: an equilateral triangle, a square, an equilateral hexagon and an equilateral octagon (24:3 = triangle/augmented triad; 24:4= square/diminished arpeggio; 24:6= hexagon/whole tone scale; 24:8 = octagon/ new symmetric scale).

This way we have reconstructed in the context of 24 system notes, all the equilateral tonal functions that has been defined in the 12 note system by Slominsky. Of course, the tonal functions are the same *from an abstract point of view*: the music intervals that define are similar, always composed by the same number 3,4,6.
But of course the tonal functions (and the related scales) are different (because the number of intervals into the ordered sequence is different):

12 notes system : [3,3,3,3]; [4,4,4]; [6,6]

24 notes system : [3,3,3,3,3,3,3,3]; [4,4,4,4,4,4]; [6,6,6,6]

This important property is satisfying: each scale into the 12 note context is contained into the scale in 24 context generated by the same divisor.

The new specific octagon symmetric scale corresponds to an *eight-tone perfectly symmetric scale.* It has a peculiar sounds and it is linked to the scale *step - half step or the Messiaen's second mode* which is largely used by European and American music, (both in classical and in jazz music), as example Stravinsky [22] and Diminished scale Coltrane patterns. I have not observed the usage of this scale in any kind of extra European traditional music, however it sounds very coherent since it is related to the diminished step - half step. It is a transfiguration of the common diminished scale to an equilateral symmetric scale which uses quarter tones. For this reason I have used this scale a lot in my music and compositions, precisely because of its not conventional sound and its peculiar harmonic characteristic. Furthermore I want to clarify that, even if eight tone scale does not generate tonal areas, in traditional harmony it is useful to generate some dominant chords possible to connect many tonalities. Thus octagonal scale as well is useful to assume the function of connector. It is useful to navigate through different scales and chords since it recalls the altered dominant function of the diminished scale.

4.5.2 Partial symmetries

The other type of symmetry is the partial one.
When I had to decide which partial symmetry figures I wanted to obtain tonal functions,

[22]The importance of the octatonic scale in the music of Stravinsky was originally proposed by Berger. See the article:
Arthur, Berger. "Problems of Pitch Organization in Stravinsky." *Perspectives of New Music, Vol. 2, No. 1.* (Autumn - Winter, 1963), pp. 11-42. doi: 10.2307/832252 .
An interesting article that critically analyze these aspects is:
Dmitri, Tymoczko. *Stravinsky and the Octatonic: A Reconsideration.* [pdf], 2002. Available at: dmitri.mycpanel.princeton.edu/files/publications/stravinsky.pdf [Accessed 18 Jun. 2017].

therefore **I did not consider to use figures with less or more than four sides.** *All the figures with more than four sides can be reduced into more figures of three or four sides.* Such as rhythms in music are made up of combinations of two and three, I adopted the same approach for geometries.

So, I have approached the more basic geometric figures: rectangles, diamond and isosceles triangles.

Polygons with more layers could be studied in further researches, for the moment I decided to focus on the basic figures in geometry.

4.5.2.1 Four sides figures: rectangles and diamonds

The rectangle and the diamond attend the following rule: they are made of four equal sides in pairs (a,a,b,b). Since we are only interested to study the length of sides (the music intervals), we consider them as the same type of figure.

When I inscribed a rectangle and a diamond in the circle of 24 points I had thus to create the following equation 2a + 2b = 24 or, a + b = $\frac{24}{2}$ = 12 in order to generate figures which could have covered all the range of the octave. Essentially it is needed to find all the couples of natural values from 1 to 12 that added up makes 12.
These are the pairs: (1 + 11); (2 + 10); (3 + 9); (4 + 8); (5 + 7); (6 + 6).

On each of these couple of values, it is necessary to compute the related *absolute tonal function* (the cumulative function of the set of music intervals, putting together all the tonal function that are defined by one permutation of the ordered sequence elements, according to the Absolute similarity principle). The computation of an absolute tonal function has been made using the geometrical approach previously explained: drawn together all the constituent figures into a common picture.

In the next graphic there are presented all the four sides figures where 2a + 2b= 24 and a =5 and b=7, and then is computed the union, i.e. the absolute tonal function.

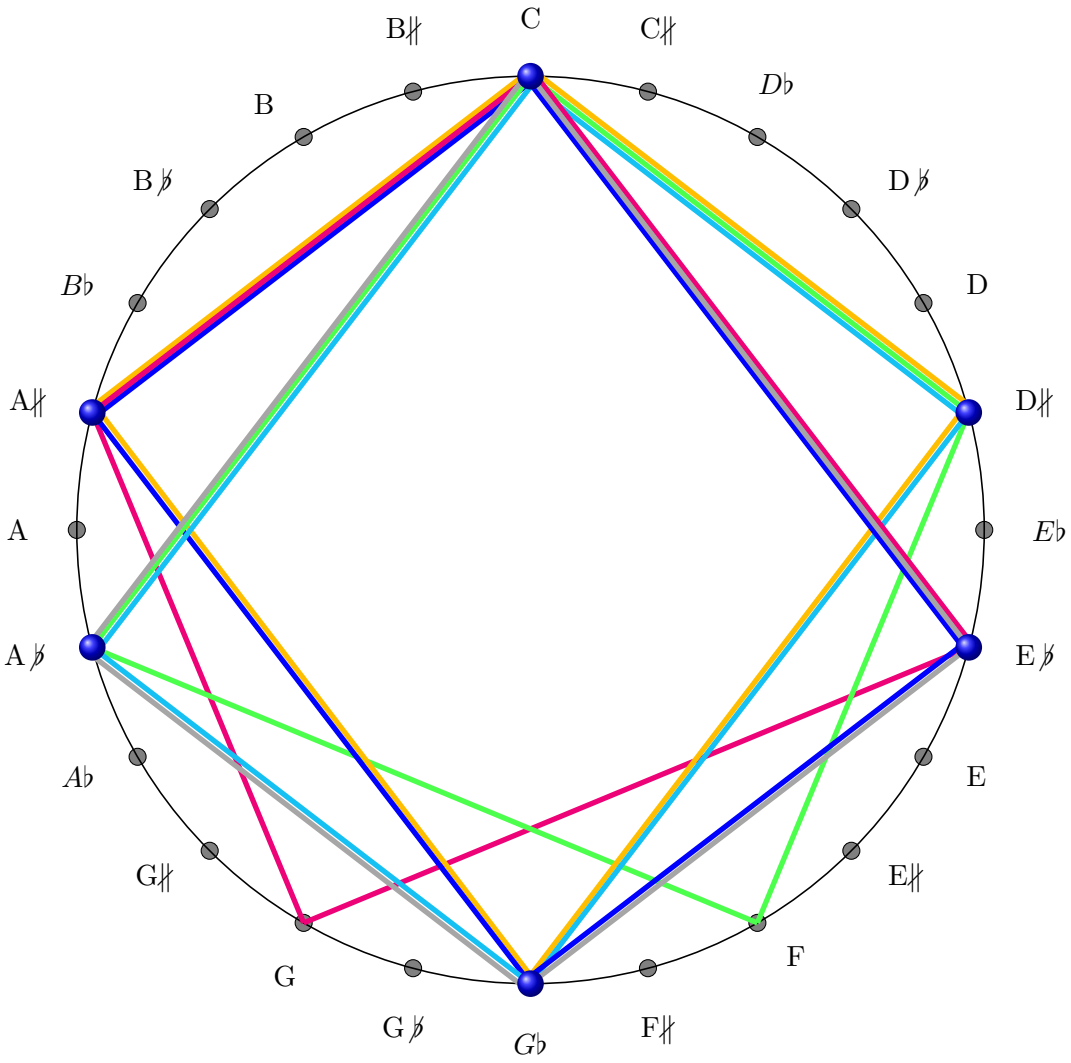


Figure 4.8: Rectangles [5+7]

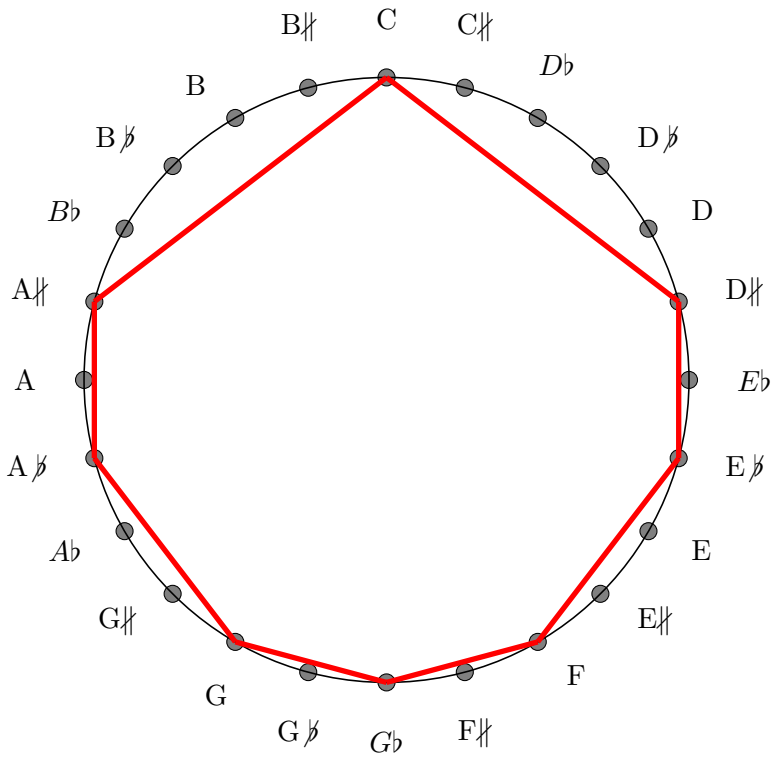


Figure 4.9: The Absolute function for [5+7] rectangles

To avoid a useless complication in reading, the computation of the absolute tonal function for all other couple of values is set in the appendix (see chapter A).

In the following list, they are reported the results (the column source reports the context of computation for the absolute tonal function, that is the geometric rectangles that have been put together by the Absolute similarity principle): It is important to highlight that

Absolute Tonal Function	**Scale on C**	**Source**	**Num of elements**
[1,1,9,1,1,9,1,1]	{C, C𝄪, D♭, F𝄪, G♭, G𝄫, B, B𝄪}	[1+11]	8
[2,2,6,2,2,6,2,2]	{C, D♭, D, F, G♭, G, B♭, B}	[2+10]	8
[3,3,3,3,3,3,3,3]	{C, D𝄫, E♭, E𝄪, G♭, G𝄪, A, B𝄫}	[3+9]	8
[4,4,4,4,4,4]	{C, D, E, G♭, A♭, B♭}	[4+8]	6
[5,2,3,2,2,3,2,5]	{C, D𝄪, E𝄫, F, G♭, G, A𝄫, A𝄪}	[5+7]	8

Table 4.2: Absolute tonal functions for four side figures.

I did not consider here the couple [6 + 6]. This is because in this case a = b and so the defined geometric figure is an equilateral figure, the square, or a diminished arpeggio, that has already generate and analyzed in the context of perfect symmetry (see figure 4.7 on page 72).

4.5.2.2 Three sides figures: triangles

The following geometric figure I have considered is the isosceles triangle: it has three sides instead of four but, as for the rectangle, it has two equal sides.

It can be described as 2a + b.

As I did before we need to find now all the figures that correspond to this relation formula: 2a + b = 24. [23]

I got these couples of values:
$[\frac{2}{2} + 22]$; $[\frac{4}{2} + 20]$; $[\frac{6}{2} + 18]$; $[\frac{8}{2} + 16]$; $[\frac{10}{2} + 14]$;
$[\frac{12}{2} + 12]$; $[\frac{14}{2} + 10]$; $[\frac{16}{2} + 8]$; $[\frac{18}{2} + 6]$; $[\frac{20}{2} + 4]$; $[\frac{22}{2} + 2]$.

Since the triangle has only three sides the number of figures are fewer compare to rectangles; consequently also their results and the ways to use them.

I left out the pair [16/2 + 8] because it describes an equilateral figures, that has already generate and analyzed in the context of perfect symmetry (see figure 4.7 on page 72).

As it can be observed, with triangles the possible permutations are only three and the number of points it comprehends in the circle are nearly always five; *this means that each triangle represents a pentatonic scale.*

[23] In triangles I had to find a couple of values {a,b} where a has to be possible to divide by 2. So I have written each couple as $\frac{a}{2}$ + b.

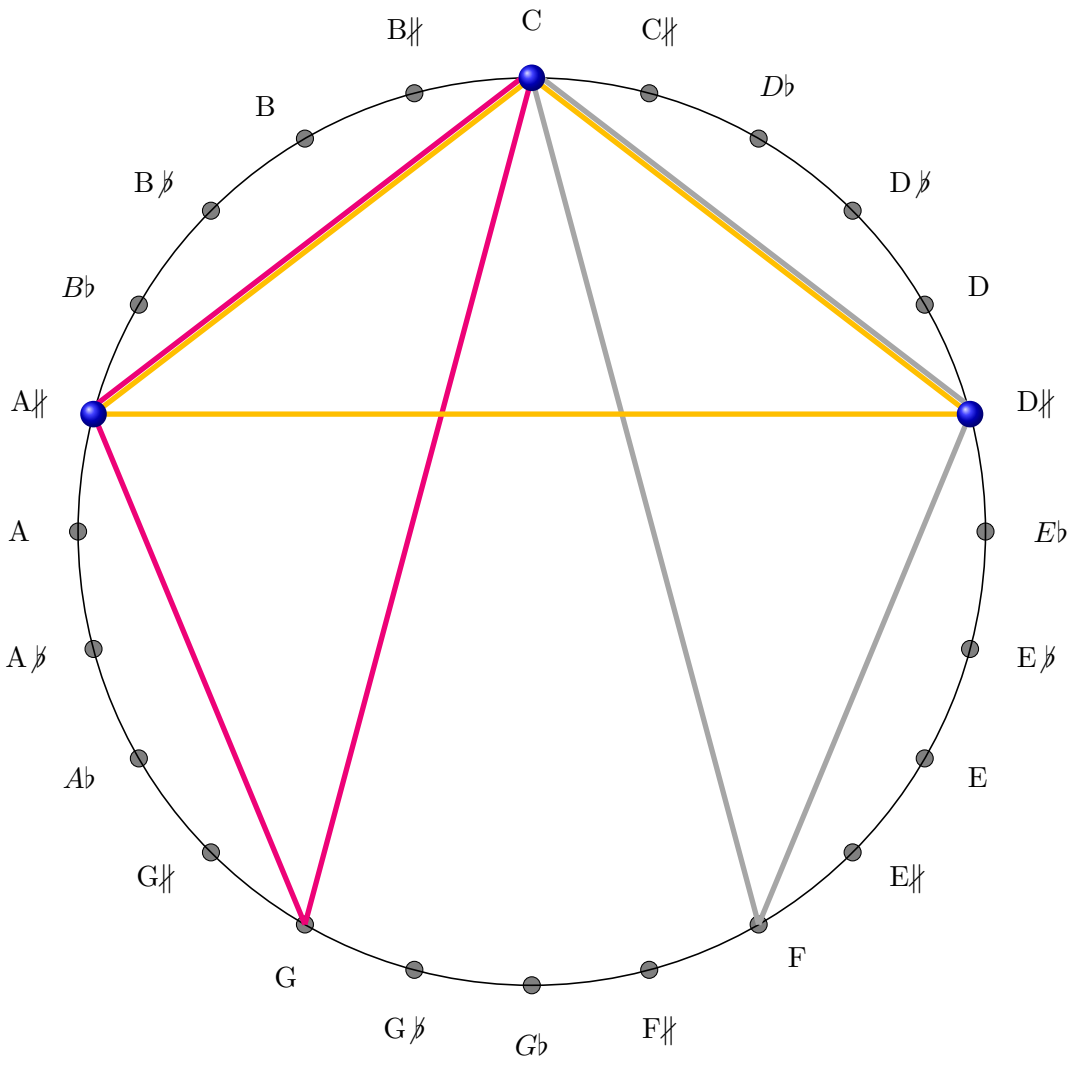


Figure 4.10: Triangles $[\frac{10}{2}\ +14]$

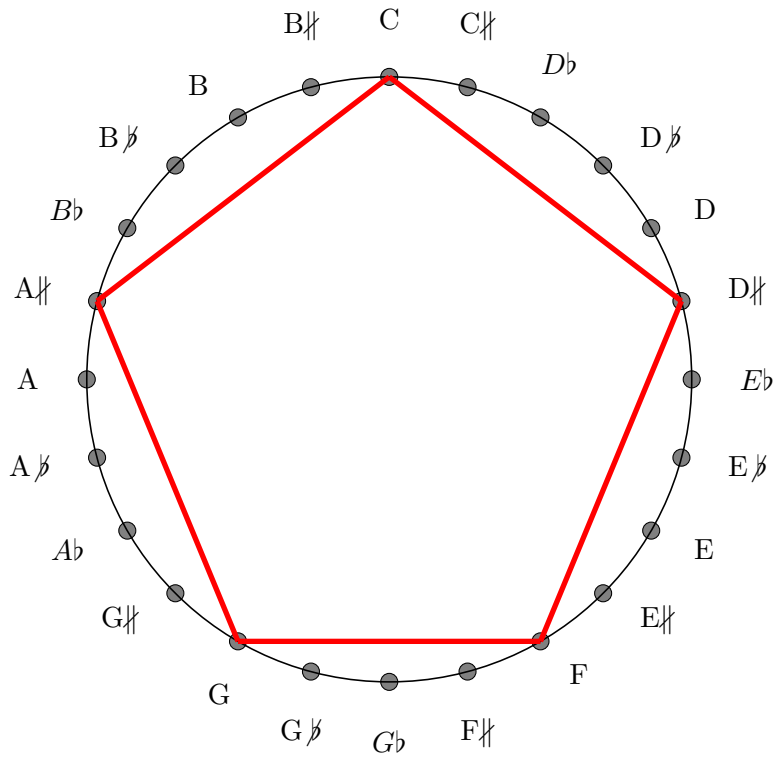


Figure 4.11: The Absolute function for $[\frac{10}{2}\ +14]$ triangles

Again, to avoid problems in reading, the computation of the absolute tonal function for all other couple of values is set in the appendix (see chapter A).

In the list, they are reported the results (the column source reports the context of computation for the absolute tonal function, that is the geometric triangles that have been put together by the Absolute similarity principle): We observe that functions $[\frac{12}{2}\ +12]$ has as

Absolute Tonal Function	**Scale on C**	**Source**	**Num of elements**
[1,1,20,1,1]	{C, C𝄪, D♭, B, B𝄪}	$[\frac{2}{2}\ +22]$	5
[2,2,16,2,2]	{C, D♭, D, B♭, B}	$[\frac{4}{2}\ +20]$	5
[3,3,14,3,3]	{C, D𝄫, E♭, A, B𝄫}	$[\frac{6}{2}\ +18]$	5
[4,4,8,4,4]	{C, D, E, A♭, B♭}	$[\frac{8}{2}\ +16]$	5
[5,5,4,5,5]	{C, D𝄪, F, G, A𝄪}	$[\frac{10}{2}\ +14]$	5
[4,4,12,4]	{C, E♭, G♭, A}	$[\frac{12}{2}\ +12]$	4
[7,3,4,3,7]	{C, E𝄫, F, G, A𝄫}	$[\frac{14}{2}\ +10]$	5
[6,3,6,3,6]	{C, E♭, E𝄪, G𝄪, A}	$[\frac{18}{2}\ +6]$	5
[4,6,4,6,4]	{C, D, F, G, B♭}	$[\frac{20}{2}\ +4]$	5
[2,9,2,9,2]	{C, D♭, F𝄪, G𝄫, B}	$[\frac{22}{2}\ +2]$	5

Table 4.3: Absolute tonal functions for triangles.

result a four notes *absolute tonal function*. This function therefore is not able to generate tonal areas, we consider it an *arpeggio*; more precisely the *diminushed arpeggio*. By the way, this scale (as the square 6 + 6) has been already encountered and analyzed in the context of the perfect symmetry (see figure 4.7 on page 72).

It has now been exposed finally the generative process of the method. Thanks to geometrical principles they have been founded many combination of notes.

I will not list here all the possible combinations into a table, since that would be a huge amount of scales. By the way I have personally written down all the lists, and I have gone through them in order to understand which one could have been useful and which properties I was searching for in order to go through the qualitative process of this method, they cam be found in the addendum.

Of course I will not leave the reader high and dry: the scales I have used in the composition process are listen and discussed further in chapter 6.

Said this, now it has to be found some criteria to select notes among these results and categorize them into a useful context. It has to be analyze the faculty to generate tonal areas and the thoughts that have been considered to finally chose some scales.

I want to anticipate already here that in the following section of the method it will be operated a further generative operation, after a qualitative selection of the material.

In particular, pentatonic scales here obtained will be re-constructed according to a not geometric process, gluing again this method to the traditional music theory principles.

Chapter 5

Methodology: qualitative process

We now go further with the treatise of the methodology: after the generative process we now approach the qualitative one.

The generative section of the method has provided me a lot of interesting material. I got a lot of tonal functions and more generally set of coherent notes; however I had to choose in between all the relationships I got from symmetries. Furthermore the previous section of the method has followed only geometrical principles and thus it has not generated complete results. It is now needed to extend the process leading it back to a music theory approach. Thus, I worked on finding several ways to interpret the group of notes I got and then decide which one was relevant. In other words, I had to decide **which strategies I had to use to generate valuable scales from the material.**

I now go through them, I explain these strategies I used to grasp further material from rectangles and triangles. If previously we have mainly faced mathematic and geometries principles, here both them with semantics and pure music theory are gathered together.

5.1 General principles

I here discuss the main principles I defined into the definition of a method to choose notes in between the multitude of results I got in the previous section of the method.

Furthermore, in this section I focus my attention on the issues regarding the capacity of a tonal function to generate tonal areas.

As explained in chapter 3 and 4, the additive modular operation allows us to determine a tonal function defined as an ordered sequence of intervals.

This tonal function is still bare of tonal areas; the elements have been defined, but they still have to be organized through the partition in tonal areas, in order to complete the operation (we are aiming to) of creating some scales which are able to generate grammar and syntax in horizontal harmony.

Thus in this chapter there are presented the processes I have adopted to select among the relations I have obtained thanks to geometrical processes into the generative section of the method.

A first selection has been made through others geometric and generative processes; I have created some operations with which I have determined a selection of notes into the *absolute figures* I got from the previous section. This first part of the qualitative process, as the reader will observe, has still some generative component (I have *generated* a selection of notes through figures, through geometric principles). It has therefore occurred before a reflection upon the problem of tonal areas. This first part can be considered as a continuation of the generative process into a qualitative selective context (the geometric principles remain therefore unvaried).

Nevertheless a second section part of this qualitative process takes moves from some reflections upon tonal areas.

It is thus clear that it is very important to define now to property of tonal areas, how they are constituted, which axioms they have to attend in order to be considered valid.

I have to admit that it is hard work to propose some criterions to define precisely what is a tonal area and when we can agree upon the fact that some tonal areas are *valid* and some are not. Thereby I will here propose to the reader a very extensive definition of tonal areas; I will here postulate some generic axioms that the tonal are can not refuse to attend rather than propose some rules it has to attend to be considered valid.

As first general principle, I want, claiming again George Russel, to firstly remind that tonal areas imply diatonicism; they imply some scales in which we have to be able to locate some kind of double tonic.

Said this I now define a list the properties that a tonal area can not unattend:

- Every area has to contain elements which are respectively uniform and coherent. The class defined by the tonal area must be satisfied by every element which belongs to it. It also means that, according to this homogeneity, the elements belonging to a tonal area must be possible to be superimposed according to their common quality.

- Every element of the tonal function has compulsory to belong to at least one tonal area. No element can be excluded and rather more than one element can belong to one tonal area (as explained in section 4.1, tonal areas define a *partition*).

- They have always to exist all the three tonal areas; it is not possible to have only two or one of them.

- A scale has to contain at least 6 notes in order to generate tonal areas since the tonal areas have to contain at least two elements each one. It is not possible to have three tonal areas with on element each; it is not possible to have one class acting on one single element.

I met these axioms reasoning on the already affirmed tonal scales as well as trying to define tonal areas in completely random invented sequences of notes.

These principle define tonal areas possible to generate chords and, more generally, they define rules regarding the minimum number of elements. As the reader might have noticed, there is no rule saying us *" what specific quality have the notes to attend in order to be possible to be gathered into a tonal area"* or in other words: *which quality is defined by the class Tonic, Dominant or Sub-Dominant.*

I did not undertake this discussion since it was very hard to answer the problem, if is there any property possible to be defined, it belongs to the faculty of our musical ear. I do not investigate here what is it, how can it be described and how do I think that it is somehow universal and not belonging to the interpretation of the single listener. This issue could be developed in further studies.

However these axioms allow us to define some of tonal areas, at least to define some unescapable rules they have to attend. Moreover the research of tonal areas into a tonal function probably belongs more to a certain type of attitude in considering the scale, rather in some special qualities of the notes.

Saying this I mean that the way we intend the scale shapes its quality. I mean that when we define a syntactic scheme to interpret our tonal function, automatically our tonal function becomes possible to generate tonal areas and therefore a grammar.

Furthermore, I want to propose a description of music where the properties of the language are not enclosure into the notes itself, rather into the adoption of a syntactic schema which is set over them. Therefore it is the syntactic schema which has to attend some properties, not the notes. Then the notes can be inscribed into class defined along with the syntactic schema, and therefore acquire meaning according to it.

These affirmations seems to confirm the juxtaposition of multiple layers of meaning in languages and music language as part of them. While the notes itself will have a meaning for themselves, the syntactic structure built over them will respect some other properties.

Furthermore I want to claim already here the work done in section 5.5 since I believe it implies a reflection upon tonal areas interesting to be discussed preliminary here.

We have previously defined (in previous chapter as well as in the just passed lines of the text) as valid tonal function whatever function which is constituted at least by five notes. Nevertheless this five notes tonal function can not generate tonal areas since (as stated in the axioms) tonal areas must contain at least two elements each.

What does this mean? We consider of course as valid pentatonic scales an therefore five notes tonal functions; however we can not accord them the ability to **generate** tonal areas. How is it possible that in tonal music (I especially think about jazz music in this particular assertion) pentatonic are very useful even though they are not generating tonal areas?

Furthermore we can recognize some tonal functionalities in the set of a pentatonic, but we can not grant them the capability to generate tonal areas themselves: they have to rely on a bigger generative scale.

It means that we always consider pentatonic as reduction, as partial selections, of a bigger scale. Even if we do not deduce a pentatonic from a bigger scale, we can do the same process in its opposite direction: to deduce a bigger generative scale from a pentatonic.

This means that when we aim to identify tonal areas into a pentatonic we have to drive it towards its generative scale; the tonal areas generated by it will apply as well on its reduction.

This assertion implies that there are classes that apply on set of notes which did not generate them. It implies that we might drive set of notes to a bigger schema which can explain them and organize them through the concept of tonal area.

Every time we will listen to some system of notes, we will automatically conduce it towards a context in which they can belong to tonal areas.

This also implies that when there is a limited number of systematized notes, the listener is always torn between many bigger system that can express them. As stated previously, elements are possible to be gathered by many classes with a sort of dialectic process.

Every element can possibly satisfy a property necessary to be included into a class and a class can give any element a peculiar property it had not before to belong to it.

Thus music *signifié* is defined by the act of generating class of interpretation of the elements, which therefore acquire a sense, a meaning, a peculiar property according the syntactic system this way created.

Furthermore, the act of listening music consist in the act of trying to deduce classes of signification relating the notes into schemas which are able to determine their horizontal developing. Thus listening (or receiving and de-coding the message) means to try to predict the developing of the lines, according to the definition of tonal area classes.

Going on with the reflection upon tonal areas property, I want to propose a procedure I have developed towards the calculation of tonal areas into melodic fragments which might be very stimulating and useful, I think.

5.2 Tonal areas calculation, syntactics trees

While I was reasoning on the definition of tonal areas and on their functionality into music, I bumped into an interesting book from the researcher Aniruddh D.Patel [1].

[1] Aniruddh Pathel. *Music, language and the brain*

He analyzes musical phenomena with the lens of cognitivism, hypothesizing and discussing a common brain area involved into the comprehension and elaboration of music and verbal language. He therefore starts analyzing the phonology of verbal language, arguing how the choice of determinate sounds determines part of the meaning of the verbal context. He writes there already interesting points which seem to valuate this research (phonology of course does not regard the topic of this work, *au contraire* here we are interested in syntax of sounds which can be rather considered the other side of the medal). He writes [2]:

[...] pitch is the primary basis for sound categories in music (such as intervals and chords), timbre is the basis sound categories for the speech.

It can be found already previously in the text some quotations of Patel which seems to recognize (from a cognitive standpoint) the special characteristic of tonal music organization, it strength and its similarities with the verbal language. However my reflections started while I was reading a very interesting section of that text, called *syntax*. There Patel undertakes a reflection upon the syntactic properties of music, proposing us what are the similarities to verbal language and what are the differences.

In Patel's book that section starts with a critique to Bernstein *The unanswered question* (already cited book in this text), in which he discusses the Bernstein ideas regarding the possibility to identify the *parts of the verbal speech*, its syntagma into music analysis.

He proposes the approach of Lerdahl and Jackendoff (1983) which use syntactic trees to undertake the analysis of music scores such as J.S.Bach chorals. However, he finally concludes that there is a deep difference between verbal language syntax and presumed musical language one. In particular he affirms (p.267):

The key to successful comparison is to avoid the pitfall of looking for musical analogies of linguistic syntactic entities and relations, such as nouns, verbs, and the constituent structure of linguistic syntactic trees. Once this pitfall avoided, one can recognize interesting similarities at a more abstract level, in what one might call syntactic architecture of linguistic and musical sequences. These includes the existence of multiple levels of combinatorial organization, hierarchial (and recursive) structuring between elements in sequences, grammatical categories that can be filled by different physical entities, relationship of structure versus elaboration, and context-dependent grammatical functions involving interdependent relations between elements. These similarities are interesting because they suggest basic principles of syntactic organization employed by the human mind.

Even if I find myself very close to this position, I might move some critiques (or rather clarifications) towards it.

First of all I want to remind to the reader the different ambit that is taken by this research into the analysis of music. Here there is not discussed the music phenomena from a cognitive standpoint; here we are not interested into the comprehension of the processes

[2] Aniruddh Patel. *Music, language and the brain*, p.9

which take part into humans mind when they perceive music. Rather we want here to analyze music as a communicative phenomena, we want to understand its code, how is it possible to stream information and therefore how is it structured.

We are thus not interested into the answer of the question: " can everyone perceive music the same way? ". Or in other words, we are not interested to set a standard of interpretation of music basic structure. We just take for grant that there is a way in which music is structured and we try to deepen the way it is constituted proposing a comparison to the verbal syntactic structure.

Made this premise I want to critique the ambit of the Patel previous assertion.

When he says that music can not be approach with linguistic items such as syntactic trees he is partially true, but at the same I think it is needed a reduction of ambit to analyze music syntax into a useful way. If we consider music as a global phenomena there are too many parameters and juxtaposed classes to be considered. Even if we reduce music to its pitch organization (as we do here), there is still to be done a further distillation of the essence of pitch organization.

As said previously into the research, pitch organization is both constituted by phraseology and harmonic sense. We can divide this two aspects into *melodic sense of a line* and *harmonic sense of a line*. While melodic sense, or phraseology, involves the organization in time of music (rhythm); harmonic sense can be considered *synchronic*.

While we can only observe melodic patterns into a time evolving frame, we can study *strings of musical syntagma* into a synchronic approach. It is here therefore presented the difference between diachronic and synchronic approach into semantic analysis which has been discussed by Barthes and Eco. Since semantic approach (which now coincides with our attitude to approach the issue) tries to identify some *structures*, beyond the languages; it proposes a synchronic approach: it describes a static world of rules in which the diachronic develop of language fit itself within a dialectical process.

Thereby our analysis towards the individuation of syntax principles in music **must be limited to the harmonic analysis of lines**; it has to exclude rhythm, larger form, timber and so on.

When we limit our ambit of analysis to only the harmonic properties of lines, I believe that we might use syntactic trees and we might be able to individuate grammatical functions into syntagma. I know that this is a strong assertion, thus I will argument it here in the hope to propose interesting points of reflection.

The syntagmatic analysis involves a binary selection of possibilities through all the passages of the speech, and it demands an unitary function of the grammatical item defined (a nominal syntagma must always mean a noun functionality and so for the other ones.). These points are the hardest to be solved in our analysis of music, but let me propose some examples in which I discuss the procedure and argue time by time my assertions.

Let us suppose to analyse the following simple line:

C;D;E;F / G.

Our analysis consist into the individuation of the possible tonal movements through the notes and therefore it begins with the individuation of tonal areas.

The first step of the procedure is to individuate the most probable scale to which this fragment could belong.

I already stop here before to proceed and I recall the definition of information and code according to Eco [3]

Eco explains that we can consider to have an *information* when we have message which tells us, starting from a situation of *equal-probability*, how a future event is going to occur. For example when we flip a coin there is the same probability to obtain one face or the other; there an information when we are able to reduce these probability, when we are more able to *predict the developing of the situation.*

I quote Eco [4]:

[...] the code represent a system of probabilities juxtaposed to the equal-probability of the starting system, in order to be able to dominate it communicatively.

Furthermore, every step of choice has to be made following the process of *opposition.* All the time the receiver will pose questions which are possible to be answered with A or *not A.* We can then assume according to Eco that there a stream of information when it is defined a code which allows the receiver to make better bets on the developing of the speech.

Going back to our procedure, we now say that our attempt is to define a system that proposes the most probable or coherent develop and music, thus we are aiming to identify all the possibilities into the harmonic development of the lines and determine which one is the most probable.

We assume that this can be considered the core of music syntax: the ability to predict the *most probable* develop of the harmonic functions in lines (nota bene: not the only possible, but the most probable).

Therefore the first step is the to find the scale system which can better explain that line of notes or, in other words, the scales which can contain the most of those notes.

Since our line is constituted by five notes and we are working into a tonal and diatonic

[3] Umberto Eco. *La struttura assente.*
[4] Umberto Eco *La struttura assente*, pp. 65 .

context, we miss only two notes to define the scale, only two degrees. Particulary we miss: A and B (which therefore could be Ab; Bb or A♯ or B♯). Thus we can then identify the scale of C major and F major and no other ones.

This one represent the first step of the syntactic procedure: our harmonic quality of the line is telling us which are the context of interpretation to consider in order to explain it.

C major and F major are *equal-probable* between them, thus they are considered both the same valid. We now start our analysis in C major. According to the rule previously defined, the three tonal areas are:

T = {I;III;VI} ; S = {II;IV} ; D = {V;VII}

Thereby we can affirm that in C major context they are fulfilled in this way:

T = {C;E;A} ; S = {D;F} ; D = {G;B}

Since our attempt is to determine the *harmonic properties* of this line, our code makes use of another item. Every note into the system is possible to be part of a chord (or the other way around, it is possible to harmonize every note). When we have already two or three voices playing together, our probability calculation to determine to tonal function of the notes is less onerous. Nevertheless, when we hear a *monody*, our de-coding process tries to determine which harmonic function express the notes or in other words, to which chord do they belong.

Since we are aiming the most probable solution and not the possible one, we exclude chord bigger then triads. [5]

Computing the triads into the tonal areas framework we will obtain:

$$\mathbf{T} = \left\{ \begin{array}{lll} C; & A; & E; \\ E; & C; & G; \\ G; & E; & B; \end{array} \right\}$$

$$\mathbf{S} = \left\{ \begin{array}{ll} D; & F; \\ F; & A; \\ A; & C; \end{array} \right\}$$

[5] This is an important point: we are not trying to determine every possible path of the line, every possible contextualization of it. We are not re-harmonizing the line; we are trying to define the process according to which our ear determines to most simple one. This way we could define the most simple one as the *basic signifié of the line* and all the other possibilities as the tool the language uses to express meaning into that frame. In other words, while the ear determines which is the most logical developing of the line, it reacts to every variation to this *natural* path. This way we will have different levels of meaning, different possible contents regarding the harmonic structure of musical syntax. Concluding, we will clearly not deal here with *possibilities*, rather with *probabilities*.

$$\mathbf{D} = \left\{ \begin{array}{cc} G; & B; \\ B; & D; \\ D; & F; \end{array} \right\}$$

Or, more generally into a major scale tonal function:

$$\mathbf{T} = \left\{ \begin{array}{ccc} I; & III; & VI; \\ III; & V; & I; \\ V; & VII; & III; \end{array} \right\}$$

$$\mathbf{S} = \left\{ \begin{array}{cc} II; & IV; \\ IV; & VI; \\ VI; & I; \end{array} \right\}$$

$$\mathbf{D} = \left\{ \begin{array}{cc} V; & VII; \\ VII; & II; \\ II; & IV; \end{array} \right\}$$

Therefore we can affirm that our notes in the line can potentially belong to more than one tonal area according to their harmonic property; they potentially answer to more than one class. More precisely every note belongs to three different triads (it can be respectively the root, the third and the fifth).

The dots into the matrixes are meant to express the lack of the note into the missing triad.

C$\in Tonic$; C$\in SubDom$.

$$\mathbf{T} = \left\{ \begin{array}{ccc} C; & A; & .; \\ E; & C; & .; \\ G; & E; & .; \end{array} \right\}$$

$$\mathbf{S} = \left\{ \begin{array}{cc} .; & F; \\ .; & A; \\ .; & C; \end{array} \right\}$$

Then; D$\in$ *SubDom*; D$\in$ *Dominant*.

$$\mathbf{S} = \left\{ \begin{array}{ll} D; & .; \\ F; & .; \\ A; & .; \end{array} \right\}$$

$$\mathbf{D} = \left\{ \begin{array}{ll} G; & B; \\ B; & D; \\ D; & F; \end{array} \right\}$$

Then; E$\in$ *Tonic*;

$$\mathbf{T} = \left\{ \begin{array}{lll} C; & A; & E; \\ E; & C; & G; \\ G; & E; & B; \end{array} \right\}$$

Then; F$\in$ *SubDom*; F$\in$ *Dominant*.

$$\mathbf{S} = \left\{ \begin{array}{ll} D; & F; \\ F; & A; \\ A; & C; \end{array} \right\}$$

$$\mathbf{D} = \left\{ \begin{array}{ll} .; & B; \\ .; & D; \\ .; & F; \end{array} \right\}$$

And finally; G$\in$ *Dominant*; G$\in$ *Tonic*.

$$\mathbf{T} = \left\{ \begin{array}{lll} C; & .; & E; \\ E; & .; & G; \\ G; & .; & B; \end{array} \right\}$$

$$\mathbf{D} = \left\{ \begin{array}{ll} G; & .; \\ B; & .; \\ D; & .; \end{array} \right\}$$

Therefore we might now re-write our music line as a succession of possible tonal areas, like this:

C;D;E;F / G.

$$\left\{ \begin{array}{cccccc} T & S & T & D & / & T \\ S & D & T & S & / & D \end{array} \right\}$$

Please note that the verticality of this graphic represent the double possibility of every note to belong to multiple tonal areas, as evident from before.

Before to proceed we now have to better define the movement boundaries defined by tonal areas.
Firstly we say that:

D $\longrightarrow$ T (Perfect cadence) ; S $\longrightarrow$ D (Perfect cadence: preparation)

Also we of course say that:

D $\leftrightarrow$ D ; T $\leftrightarrow$ T ; S $\leftrightarrow$ S

Also we define the functionality of S such as a double functionality (depending from the context or, as the reader will observe later, from the punctuation):

S $\longrightarrow$ T (Plagal Cadence) ; S $\cong$ T (We define the SubDominant area such as *alternative Tonic area*)

We complete the commands saying that:

D $\longrightarrow$ T $>$ S $\longrightarrow$ T ;

S $\longrightarrow$ D $=$ D $\longrightarrow$ T ;

Once posed these considerations, we can go further with our syntactic analysis of the line.

We now consider some probable possibilities trying to individuate the *most probable* one, and therefore the *basic meaning* de-coding the message.

The first note can belong both to ***T*** and to ***S***, since it starts (there is no boundary coming from before) and both ***T*** and ***S*** represent arrive point, resolution and not movement (these

properties will be further on discussed more in details, for the moment let me just proceed with the reasoning).

In order to determine the second step of our line, our ear has to wait until the third one in which we can observe the double value: $\boldsymbol{T}$ and $\boldsymbol{T}$.

According to the defined rule: D $\longrightarrow$ T > S $\longrightarrow$ T we then affirm that it is more probable (or coherent) to assign the value $\boldsymbol{D}$ to the second step.

Going on with the analysis we arrive to a double equal-probable possibility. We can observe that between the fourth and the fifth step (however on the edge of the bar) we find both two valid solutions:

D $\longrightarrow$ T and S $\longrightarrow$ D.

According to the rule such that S $\longrightarrow$ D = D $\longrightarrow$ T we can assume that these two options are both equally valid and probable.

Resuming, we have now described our line such as:

$$\left\{ \begin{array}{cccccccc} T & D & \longrightarrow & T & D & / & \longrightarrow & T \\ S & \longrightarrow & D & \longrightarrow & T & S/ \longrightarrow & D & \end{array} \right\}$$

In order to determine our two ambiguous point (step one and step five) we might consider the position of these into the phrase. The first step obviously starts the communication, therefore, even if both of the options are valid, probably the function $\boldsymbol{T}$ is the most convenient, since it better affirms the tonality in which we are.

It might be argued that if we would have chosen $\boldsymbol{S}$ then we would have had:

S $\longrightarrow$ D $\longrightarrow$ T

which is of course a great syntactic solution.

I might affirm by the way that the function of *affirmation of the tonality* into the beginning of the phrase might be stronger. However they both are valid since they both have strong arguments. In fact our ear will not perceive a big functionality difference whenever we will choose one or the other one.

On the other hand, regarding the step five I would surely affirm that the function $\boldsymbol{T}$ is the most probable.

The step five is in fact at the end of our phrase, it represent its ending point, its arrival. If the function $\boldsymbol{T}$ has a conclusive quality (and again confirms the tonality); the function $\boldsymbol{D}$

has a movement quality. Thus we will easily accept the function ***D*** on step five if the phrase continues to others elements; on the other hand we would refuse it when it represent the end of the phrase.

At this point we can therefore define as the most probable harmonic interpretation of the proposed line the following functions diagram:

$$\boldsymbol{T\ D} \longrightarrow \boldsymbol{T\ D} \not\longrightarrow \boldsymbol{T}$$

It follows that, while this line is strictly coherent to the tonal areas movement rules, the other possible lines (even if licit [6]) are not the *preferential choice.*

They will emerge as atypical choices if supported by a coherent harmonic context (they will emerge only if supported by chords); however if the line keeps being a monody, the listener will interpret the chosen one as incontrovertible.

It is clear that music (limited to its harmonic feature) is a language which carries information with a very high value. [7]

The listener has to consider a lot of possible variations of the message but I truly believe that at the end he or she can definitely determine the most probable solution and then the meaning of the information. This could explain why music is so ambiguous to be determined as syntactic structured language but why at the same music nobody can deny to music some structural skeleton.

Furthermore we can now hazard an individuation of grammatical part of the speech in music syntax, analyzing the property given by the definition of tonal areas into determinate sections of the line (that we will call here syntagma as in the verbal language).

Similarly to the verbal grammar we identify firstly this first two functions:

Sentence = Nominal syntagma + Verbal syntagma.

S = Np + Vp.

If we shift this formulae to music syntax we might affirm that:

Musical phrase = Tonic + Dominant.

Mp = $\boldsymbol{T}$ + $\boldsymbol{D}$

Thus, since I propose a parallelism between Vp and Dominant, we affirm that the Domi-

[6] Again, these rules are not necessary to the constitution of music, they just define a preferential order of interpretation of the lines, giving a de-coding framework.

[7] Eco defines the value of an information determining how many oppositional choice have to be made in order to de-code the message.

nant area involves *movement*, action and *consequentiality.*
Whenever it will occur a Dominant, it will claim a resolution or, from another point of view, it will determine a **relation between elements.**

As the ***Vp*** determines relations between nouns, adjectives and so on; the ***D*** determines a relations between ***T***; ***S*** and other ***D*** as well.

Same as it is in verbal syntax, it can not exist a verb without a subject; in music syntax it is ambiguous to place Dominant without other non-Dominant chords.

Differently from the verbal language in music it will never exists something *wrong* [8]; but once again we are aiming to the most simple, then the core of the rules of coherency in tonal music.

If the ***D*** can be compared to the ***Vp***, definitely ***T*** and ***S*** can be compared to ***Np***.

The ***T*** and ***S*** can surely be compared to the function of subject and object of the verbal speech: they are the elements put into relation by the operative element ***D***.

We observe thus that it can be somehow hazarded a comparison between verbal syntax and harmonic syntax of pitches in tonal music.

Of course when we go deeper into the grammatical classification of the logic functions of the predicates, the comparison becomes useless and deviant. By the way we can identify further classification into the musical syntax, we can accord different functions to the tonal areas depending on the role they attend into the analysis of a musical phrase.

I list some of them:

The function ***S*** when it is intended as S $\longrightarrow$ D can be considered a different movement predicate we will name: ***Vp(SubD)***.

The function ***S*** can of course occupy the function ***Np*** with ***T*** even if with a different attitude.

Furthermore again the function ***S*** when it is intended as S $\longrightarrow$ T can be considered as a softer movement predicate we will name: ***Vp(SubT)***

We will thus have: ***Vp(Dom)***; ***Vp(SubD)***; ***Np(T)***; ***Np(S)***; ***Vp(SubT)***.

Of course the more we complicate the phrases, the more we will need more items of analysis, such the ones the explain modulations, more complex chords, altered chords. I will not talk about them here, they could be studied in further researches.

[8] As said in the introduction, we might affirm that there are wrong messages in tonal music, but only when it happens a wrong juxtapositions of contents. For example if we juxtapose a Tonic functionality to a Dominant one and so on.

Before to leave this I believe extremely interesting parenthesis, I introduce another concept.

We have until here indeed gathered all the elements possible to be categorised by a class of tonal area, anyway, even if they share the same property, they are different. Thereby when say that:

$$\mathbf{T} = \left\{ \begin{array}{lll} C; & A; & E; \\ E; & C; & G; \\ G; & E; & B; \end{array} \right\}$$

We are putting together C triad with E minor triad and A minor triad.

These elements are different but similar according the class ***T***. We will then say that:

T = {C ; Emin ; Amin} $\longrightarrow$ C $\equiv$ Emin ; C $\equiv$ Amin ; Emin $\equiv$ Amin.

This way we introduce a further nuances into the syntax: the choice of one or the other harmonic unity belonging to a tonal area determines a different specification of the phrase, a further detail in the message.

We could compare this the adjectives in verbal language. They are always correlated with a noun and they act on a quality which does not corrupt the main quality of the noun to be a ***Np***.

Also the choice of different elements belonging to the same tonal area will not change the quality of the element to belong to that tonal area, but it will shape it according to a side parameter. Furthermore it will never be possible to have an *Tonal-adjective* or ***Adj(T-S-D)*** without having a ***Np(T-S)***; exactly as in the verbal syntax.

We can thus assume that the progression:

$$\left\{ \begin{array}{ccccccc} Cmaj & G7 & Emin & Fmaj & Amin & Dmin & Cmaj \\ I & V & III & IV & VI & II & I \end{array} \right\}$$

even if it is described by the sequence:

T ; D $\longrightarrow$ ***T ; S*** $\longrightarrow$ ***T ; S*** $\longrightarrow$ ***T)***

The syntagma ***S*** $\longrightarrow$ ***T*** in steps 4-5 is different from the syntagma ***S*** $\longrightarrow$ ***T*** in steps 6-7, since the ***Adj(T)*** of step 5 differs from the ***Adj(T)*** of step 7.

The listener will therefore perceive the persistence of the class ***Np(T)*** but the varia-

tion of the other correlated class ***Adj(T)***.

Going back to our example phrase (C;D;E;F / G), we might proceed with the same analysis according to the key of F major. By the way I do not think this is the proper place to extend this topic. The operation I have proposed in C major is enough to argument the reasoning just proposed.

Concluding, the reader can observe that there are actually a lot of similarities between verbal syntax and musical syntax, contrarily to what affirmed by Patel.

I will not go longer, further researches towards this approach will be pursued, but in order to remain focused on the goal of this research I will now move on to the explanation of the qualitative process I have adopted into my method. I now divide the first step of the qualitative operation into three sides figures and four sides figures.

The first step into the qualitative section is still geometric and somehow generative. Later on it will be faced a selection which starts from the previous reflections on tonal areas and which glues the methodology to music theory.

5.3 Triangles

As said before, *triangles cumulative figures nearly always generate a five vertexes general figure* (there is a couple of exceptions as it might be observed in the list previously exposed).

Therefore, the triangles (each couple [2a + b]), generate each one, one specific pentatonic scale with its own related modes; therefore *five pentatonic for every absolute figure.*

Thus I did not face here the issue regarding the choice/limitation of the material; *au contraire* I elaborate later a system to expand the results coming from the triangles (according the previously exposed process).

Since five is the smallest number of elements required to generate tonal functions, there was not other strategies possible to be adopted into the triangles cumulative figures.

Therefore I simply considered the cumulative figure already as a full-working pentatonic, without processing it.

Many results led back to some very common pentatonic scales. For example, among the pentatonic generated through this method we can found for instance in this values $[\frac{20}{2} + 4]$ the major pentatonic (R, 2, 3M, 5, 6M) which is very common in all music. [9] Moreover in the triangle $[\frac{8}{2} + 16]$ we can found a pentatonic belonging to a Lydian dominant scale (R, 2, 3M, ♯4, 7b).

[9] Whenever values a and b are even the result lead back to twelve notes schemas since 2 quarter tones = 1 Semitone .

Furthermore all the results coming from the triangles can be found in addendum, where all the figures are listed.

Since there was not a great selection work to be made, there is not so much more to be said in this section.

Anyway I remark again that, when I worked on these pentatonic, I made further reflections upon the ability of a scale to clearly individuate tonal areas. Since I was interested to deduce a full-working tonal context I tried to deduce bigger scales from the pentatonic I got. I applied thus, a particular process to deduce diatonic scales from pentatonic [10] which will be explained in the section 5.5.

5.4 Four sides figures

When I had to use material coming from four sides absolute figures, I had much more possibilities comparing to the triangles since the generated vertexes are much more. I had to find thus some patterns to organize the notes, a process to limit the absolute figure into smaller scales.

Four sides figures, when put together according the *Absolute similarity* principle, generate always an 8 points absolute figure (except in the figure A.7 at page VIII which has the special couple of values a +2a) and so and an 8 notes tonal function.

Since **I wanted to get also 5,6 and 7 notes scales from these figures**, I had to decide some *patterns to reduce the notes from the general figure.* As mentioned in the general principles, I did not do it arbitrarily or empirically: I tried to stuck to a geometric principle in order to keep to the symmetry principle in all the steps.
I wanted to maintain the same line of thinking through the whole method, never falling in temptation to take a shortcut and rely on my instinct. I then reflected upon geometric properties of rhombs and rectangles.

Since the absolute figure always generates an eight point absolute figure and the single rectangles are always constituted by only four points (we have said as first axiom in chapter four that the considered scales must have at least five elements), I had to ideate a process to get five, six and seven notes scales from the absolute figure.

Firstly, we can observe that *each generic rectangle or rhomb can be described as the union of two triangles (divided through a diagonal)* (See figure 5.1); these triangles are called the *reduced figure* of the original one [11].

After this consideration, I started to consider some ways to make use of this property: I tried to define an operation which would have involved the reduced figures.

[10] I extended this reflections also on pentatonic coming from rectangles since it refers to every reduced scale.
[11] Please note that there are always two diagonals. They will be excluded the reduced figures which are entirely contained into the invariant figure, since they are not relevant.

Thereby I went for this idea: Firstly I considered two figures, then I have considered the merged tonal function given by the union of one figure (called the *invariant* figure) with one reduce triangle obtained from the second figure.

In other words I have put together every figure with every reduced figures obtained from the other polygons constituting the absolute figure, one by one [12].

Clearly, this operation made me find a lot of new relations and then lots of new tonal functions.

More formally, the *reduction process* in order to obtain, from a four side figure set, a set of tonal functions with a reduced number of notes, is the following: *let us select one figure which is considered invariant (not reduced) and let us mix it to (in turn) one of the two reduced figures for each other four sides polygons considered* (See figure 5.2 at page 99)).
This algorithm is repeated considering as invariant all the figures in each schema and, fixed the invariant, considering in turn all the other reduced figures from each of the other figures.

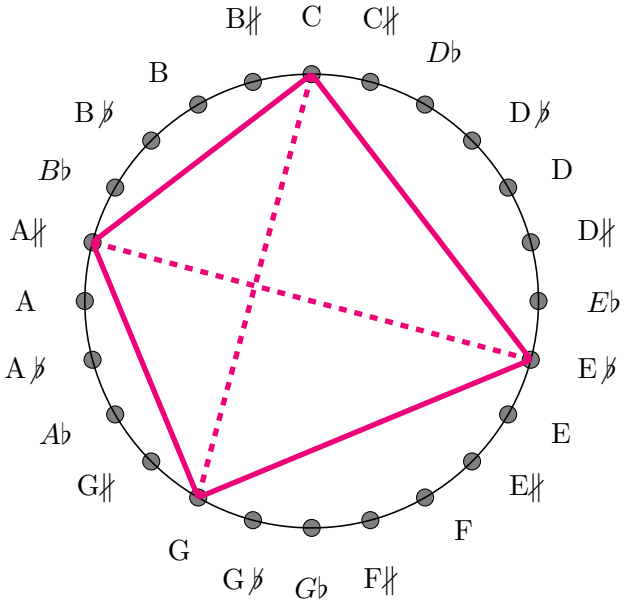


Figure 5.1: Diagonals

As an example of this process, I compute the reductions related to one invariant in the

[12] I just want to make it clear that, since the goal is to reduce the values from the absolute figure, the operation will be:
Inv + Red1; Inv + Red2 ...
Rather then:
Inv + Red1 + Red2 ...

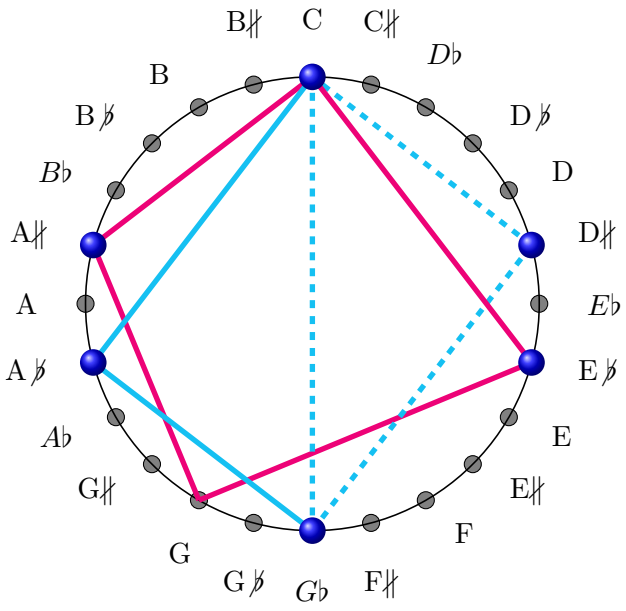


Figure 5.2: Reduction process

case of [5 + 7] tonal function (see figure 5.3 in page 100); it is defines as invariant figure the cyan figure Inv={5+7+5+7} = {C, D♯-, G♭, A♭+}.

Now I mix it with in turn all other half figures (for simplicity, as reported in the figure 5.3 in page 100, here is considered only one of the two reduced triangles): [13]
The 5 different reduced figures to consider are:

Red1 = {C, D♯-, G♭}
Red2 = {C, D♯-, A♭+ }
Red3 = {C; E♭+, A♯- }
Red4 = {C; E♭+, A♯- }
Red5 = {A♭+, E♭+ G♭ }

So (considering that there is a figure repeated twice, R3=R4):

Inv + Red1 = {C, D♯-, G♭, A♭+} ∪ {C, D♯-, G♭} = {C, D♯-, G♭, A♭+}
Inv + Red2 = {C, D♯-, G♭, A♭+} ∪ {C, D♯-, A♭+ } = {C, D♯-, G♭, A♭+}
Inv + Red3 = {C, D♯-, G♭, A♭+} ∪ {C; E♭+, A♯- } = {C, D♯-, E♭+, G♭, A♭+, A♯- }
Inv + Red5 = {C, D♯-, G♭, A♭+} ∪ {A♭+, E♭+ G♭ } = {C, D♯-, E♭+ G♭, A♭+ }

Now (of course) we discard the tonal scales with a not sufficient number of elements (less

[13] From a couple of values [a,b] such as 2a + 2b = 24 you get firstly 6 figures (considered together according to the Absolute similarity principle): f1={a+b+a+b}; f2={b+a+b+a}; f3={a+a+b+b}; f4={a+b+b+a}; f5={b+b+a+a}; f6={b+a+a+b}. So we have 5 different reduction figure to consider.

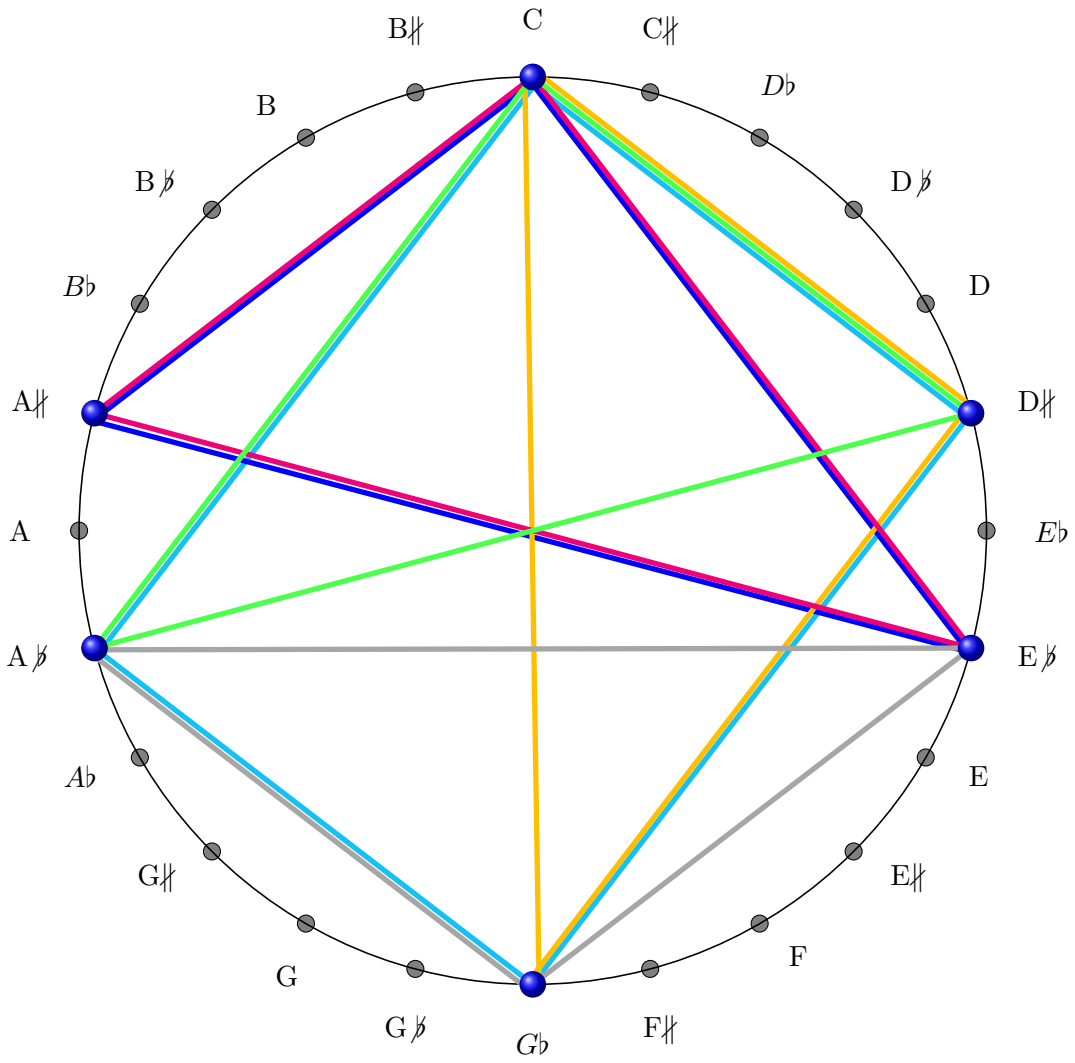


Figure 5.3: Rectangles [5+7] Reduction example

then 5), obtaining the results:

{C, D♯-, E♭+, G♭, A♭+, A♯- }
{C, D♯-, E♭+ G♭, A♭+ }

Finally, after eliminating all the repetitions, we got all the possibly combinations in a particular relation [a + b]. *This means a series of scales with five, six and seven notes that have in common the departing relation of symmetry.*

Furthermore, we can verify that the results of this procedure always generate at least a pentatonic scale up to a seven note scale.

Within this operation it is possible to obtain a lot of reduced scales according again to a purely geometric principle.

This way there are generated many seven, six and five tones scales. All of them will clearly share some common quality, since they all belong to the same generative eight note scale (the absolute figure general scale, called also *mother scale*).

I do not list here all the obtained results, since it would be really intricate to read through the text. They are set in the addendum together with the graphics they belong to.

As argued in the general principles, this step could be considered part of a further generative process, but actually it is a qualitative choice into the set of the absolute function concerning the rectangles. We here only apply a process to *make choices* in between the eight notes absolute function considered. This process of course generates new scales, but at the same time it can be considered already a qualitative choice.

Thanks to this choice process, it was possible to get a large number of scales referring to a common principle of partial symmetry, each one with differences which distinguish it.

For example I will here present some evident results.

In the figure [6 + 6] we can find the diminished arpeggio [14].
In the figure [5+7] it can be found a curious paraphrase of the blues scale with quarter tones (curiously exactly where they usually are the blue notes); R, 9♯-, 3M-, 4, 5b, 5, 6M-, 7b-.

This scale can be explained as a scale constructed on the top of the major triad (with major third) which has in addiction a major sixth borrowed from a Mixolydian mode which is often used in the traditional blues scale.

I was pretty exited certifying that these results suggested me that it could exist some relation between principles that gives order to our reality and the aesthetic principles chosen by most of the culture in the world.

Is it reasonable to think that cultural choices of some scales rather than some other else can have some kind of root into geometric relations that rule the organization of notes. Of course I can not and I do not even want to proof this assertion, by the way I take it as a validation of the geometric method I have adopted. Nevertheless, coming to this point of the research, we have generated many scales (of five, six, seven and eight notes) according **only** to a geometric process. This geometric process has been involved both into the generative and into the first qualitative part of the method.

[14] please note that every time there is a special relation between the two addends (like for instance [a + a] or [a + 2a]) the generated figures and therefore the cumulative tonal function, acquire some special characteristics as well. This is due to the special symmetric properties of figures which are generated by simple mathematical relations between their layers values. It also means that there is a strict link between geometric properties and notes schemas properties and subsequently between these two and the musical effect of scales and harmonies.

We now leave geometry entering again into music theory, making treasures of the reflections made in section 5.1 and 5.2 regarding tonal areas.

5.5 Increase number of notes: from pentatonic to diatonic scales.

As previously highlighted, the number of notes selected by a tonal function is relevant when we consider the problem to define the tonal area.

This reflection drives us to the last step of the method: the diatonic construction, and re-classification of the material. More specifically, I now reproduce all the steps I have done into the process of expanding a pentatonic towards a generative scale.

From the beginning I had to think of a scale as function able to give values to its own elements and to generate tonal degrees. Later I started to analyze each kind of scale, its properties and how they give the tonal areas S-D-T values to their elements.

As already said before symmetric scales do not generate disparities between grades. The interval is always the same and the scale sounds the same wherever we let it start (there is botany order bounder). It is not possible to identify S-D-T areas in these scales. Since I was, at this point, interested into solving the issue regarding tonal areas, as result, I omitted all the scales which are perfectly equilateral and I only focused on **how the number of elements could influence the several ways to group notes in tonal areas.**

The six and seven tones scales have their own strength and identity that cannot be generated by nothing above them: they identify tonal areas. [15]

For example the major scale does not belong to any other generative scale since there are no scales with more notes than the major one that could explain it better. The same for the six tone scale that takes shape from the rectangles [3 + 9] and [5 + 7]. Again, the six and seven tones scales are able to assign a tonal function to their elements.

On the contrary, **the pentatonic scales have a limited number of elements and they are always connected to a bigger generative scale.** The major pentatonic scale corresponds to a particular choice of notes of a major scale; its notes are inscribed in tonal functions given by the major scale.

I want to explain this in a deeper way.

In particular regarding the comparison between the Lydian scale and the Major, George Russell insists to call the major scale a *diatonic scale*, or a scale comprehending two roots, two tonics (it happens where there is an interval of perfect fourth). [16] Concerning the in-

[15] A scale is said generative scale to another one when it contains all the notes of the generated scale adding other coherent grades to the smaller one.

[16] George Russell. *Lydian chromatic concept of tonal organitation.*

terval of a fifth as one of the most important element to create consonance (See 3), Russell considered the fourth (which is the inversion of the fifth) as generator (since the backward movement in the circle of fifth) of a new root/tonic. The fourth is the principal note of the subdominant area and it can be considered as *alternative tonic* among the three tonal areas. Following Russel's theory, the diatonic major scale results as an element always moving, unstable and with a big drive through the horizontal movement. Schönberg in his work confirms George Russel (or rather the other way around) saying that the fifth is generator of consonance and the fourth is generator of movement between harmonic cadences and in scales. [17]

Here I place important considerations:
regarding the process of expansion of pentatonic, it is useful to apply the diatonic principle to create bigger scales which generate clear tonal areas.

I tried to find the diatonic characteristics in a group of 24 elements so I searched for a connection between the incomplete pentatonic scales and seven tones diatonic scales which they have been deduced.

Trying to inscribe the pentatonic in some seven tone scales as its generative scale, I formulated the opposite process to make a synthesis of the pentatonic; beginning from a pentatonic I started to add the missing degrees of a major scale, melodic and harmonic minor scales.

We take for example the pentatonic of mode 1 of $[\frac{14}{2} + 10]$: R, 3b-, 4, 5, 6M-.

This pentatonic has already set the I, III, IV, V and VI degree. It misses the II and the VII. It has the minor third, the fourth, the right fifth and the major sixth. The second should be major (either in a minor or major scale), while the seventh could be major or minor depending the original (melodic minor or dorian, second mode of a major scale). This pentatonic could sound as a choice of notes coming from a Dorian scale, harmonic scale or melodic minor scale.

I did the same for all the pentatonic scales and I found then a variation with quarter-tones of diatonic major and minor scale and its modes.

I applied this process to obtain generative diatonic scales from pentatonic both on pentatonic coming from triangles and from the ones coming from the four sides figures. It is here important to say that from the same drawing of a triangle (or the same couple of values 2a + b) I have found pentatonic scales belonging to different *diatonic fields.*

With this I mean that I generated diatonic scales which can be described as minor or major using the material coming from the same absolute figure. This means that we can identify again a multiple layer of classification: some scales can be considered similar since they respect the class quality according the geometric procedure and at the same time they can be considered similar since they attend a diatonic class property. We will interpret some element relating to one or the other classification, we will perceive coherency according to

[17] Arnold Schönberg. *Trattato di armonia*

multiple layers of meaning.

Through this new classification we introduce therefore another class of meaning, enriching the palette of possibilities into the prediction of the horizontal developing of the speech.

In this part of the process I have then re-constructed the scales I have obtained by the reduction process proposed in the last section, through a musical theory concept. Thus I leave the pure geometric field in order to approach again music theory elements I got from the application of pure geometric processes. The pentatonic obtained from the triangles and from the reduced rectangles are thus now re-constructed in order to generate tonal areas. This double classification is vital to create a double pertinency of the material, according to geometric derived scales the opportunity to be interesting into a tonal context.

For example; the couple $[\frac{10}{2} + 14]$ has pentatonic scales re scheduled into a minor scale and four SUS chord scales.

The pentatonic scales can be grouped in couples of original values but they can be re organized according to diatonic fields in several ways. I have found scales from Arabic Maquam and generally very interesting variations of traditional scales used in tonal harmony.

Again, this process to obtain generative diatonic scales from pentatonic is applied both on pentatonic coming from triangles and rectangles. Thus it represent a further re-classification of all the material I got from the geometric process. It can be considered as a particular qualitative selection which operates on all the elements of the method.

There will be a lot of re-created scales. Only the triangles generates five pentatonic each; if we add the ones derived from the four sides figures, the number of scales increases relevantly.

In the audio files attached to this research I use a lot this particular technique since those scales (this way obtained) are easily related to the tonal ones. Therefore When I had to improvise on chords sections I used those scales as variations of the more logical related one. For example on D-7 chord I played together with D dorian scale, the D quarter tones dorian related scale.

Since the re-classification generated by its application, this qualitative process can be considered the core of the qualitative section of the method. Furthermore it is the only not geometrically derived concept I adopted into this method. Thereby it occupies a special place into this research; it is the element which lead back to pure music theory.

5.6 Conclusions.

We can observe that most of the scales I got from triangles and rectangles are largely used in traditional occidental music since figures that uses couples of number divisible by two lead again to schemes made of 12 notes, since 2 Qt = 1 St.

This method has provided a huge quantity of material. Nevertheless, I have not found a single scale that was a repetition of another, which is impressive considering the huge amount of scales came up from permutation of the same 24 notes.

The way I worked allows to find scales through an only geometric approach, based especially on the search of relations of symmetry completely disjointed from all the traditional aesthetic and principles we found in the history of music. The six-tone, eight-tone diminished and the major pentatonic scale (which with its modes comprehends also the minor one) comes out simply applying this geometric formulation.

This geometric virginity, united with the last re-constructive process, is able to guarantee a truly original approach: disjointed by tradition but drove back to tonal music further on in the process.

From Pythagoras to the Flemish music of XV century until Palestrina in XVI^{th} century music and mathematics had been considered part of the same discipline. The idea of music of the spheres seems to match with these new discoveries on a different layer of consideration (not the acoustic one, rather the systemic one): even if this method has never had any contact with several world music, although it is very close to their aesthetic decisions. Moreover, an abstract method unbound from any aesthetic values as the geometric one could have found a large number of scales used all over the world.

Finally despite the result very close to the traditional sounds in music, the lack of any aesthetic values helped me to be independent in the creation of the music with this method.

Moreover I evaluate this method as valid since it led to a useful organization of a twenty-four notes context. In the following chapter I go on with an analysis and evaluation of the compositions done with the method, however the reader will judge as well the partial artistic result of the music; I have anyway to say that even if a lot of difficulties occurred, after this method I could have definitely approached quarter tone music in a both original and working way.

This method has also gone through a syntactic approach in music which I think it has been efficient and valid. Especially when it occurs improvisation, a syntactic approach is great to rule the flow into a coherent developing of music. If the sharing of syntactic rules is fundamental in group improvisation, in solo performances it is really clear how to detect the syntagma of music, how to follow the speech. Just an example; I had time to listen pretty much to Kieth Jarrett solo performances (I am now especially thinking of the ECM recording: *Creation*): there it is really simple to follow the developing of phrases and ideas (this is amplified by the *immanency* required by improvisation and freedom given by the solo format) and every change of direction in harmony is clearly affecting the quality of the music.

I pose here a closing reflection on the method.

As the reader has noticed, the whole method is centered around the organization of notes and it deals with the structure of music (as argued before, its *signifié*) considering the syntax

of music **only regarding the pitch.**

Of course they are not only notes and pitches that constitute music syntax and that thus define the *signifié* in music. Dynamics, phraseology, larger form, rhythm, timbers: they all are concurring in equal parts towards to definition of a syntactic system into the process of signification in music.

Therefore the system is much bigger and the classes of meaning which contribute to the signification are much more. They operate on the single parameters of music as well as on their relation, they can amplify the message carried by one class or contradict it.

Thus I want to conclude that music is a very complex language which makes use of a huge and intricate syntactic system of meaning. This research obviously deepen only an aspect of one of these organizations, nevertheless further studies might approach the study of music from this conception, proposing methods to compose and more generally reflect upon music that take moves from the concept of music syntax.

I now move on to the comment upon the proposed examples compositions, the reader will observe that the language will therefore become again less schematic and more colloquial as in previous chapter.

Chapter 6

Analysis of compositions and media files

They are said two things regarding the journeys. The first one says that it is never an easy travelling when the traveller aims to reach the most beautiful beach; the second one says that it is not only the yearned destination to be joyful, it is the journey as well.

Well, the reader might certainly agree that until here it was a hard way, thus she or he should definitely expect the most beautiful view, once climbed this majestic mountain. After the complicated and troubled path which has led us through the method and its devious musicological problems, we now finally reach the music: its sounds and its scores.

If I am totally sure that it was a precipitous and long rise, anyway I am not totally sure about the view from up here. If I am totally sure that the reader has longed the moment to hear the music quite badly, at the same time I can only hope that the music she or he will hear now was worth it compared to the efforts spent into the previous chapters.

Nevertheless I will not postpone the moment too long and I begin now to show the music achieved with this method.

Audio examples of scales are attached to this research as well, nevertheless I propose in this chapter an artistic application of the proposed method; I propose proper compositions and performances.

Thus, as it is clear, after the creation of the method presented I went along with the composition and performance of pieces which has been created with the material got from the method. I composed several pieces in which I experimented different techniques to use the material. Together with the explanation of the method I here reflect on the compositions and I analyze some material from the recording I am developing with Diana Torto and Michele Rabbia. [1]

[1]Diana Torto is one of the most recognized jazz vocalist in the European jazz scene. We have already collaborated in other recordings such *Corale* edited by Honolulu records in 2014. She has sung with great musicians such as John Taylor, Kenny Wheeler, Anders Jormin and many others.

The music I composed has therefore been thought to be performed in a trio setting: classical guitar, voice and percussion/electronics. In the beginning it was not easy to chose the instrumentation, since many issues were to be faced and solved. First of all it is not easy for every kind of instrument to deal with quarter tones. I initiated this research experimenting a special way of tuning my instrument which has not originally been built in order to be possible to play quarter tone music. Thus there is already a special attitude regarding my instrument, it was very hard (or rather barely impossible) to find some other colleague whom would have wished to undertake the same crazy research.

I had to find then an instrumentation that could have allowed me to utilize quarter tones without demanding too much from other instruments. Firstly I tried to collaborate in trio with double bass and soprano saxophone, however it did not work properly: music was pretty hard to come out from that constellation. There were too many technical issues, it was very hard to play (or even intonate) quarter tones on those instruments. Furthermore the music sounded as bounded into a jail of cards: we all tried to play quarter tones and the effort was so hard that we lost music.

Therefore I later on decided to undertake another road; I decided that I should have been the only one playing quarter tones into the constellation since I was the only one who could have deepen into them so much and furthermore, I was the only instrument which could have never done intonation mistakes (thank goodness, guitar has frets).

I decided then to opt for the necessary, only the strictly necessary. Since I would have been the only one playing quarter tones, it would have been problematic to share the ambit of harmony with other polyphonic instruments. Taken the harmony it remained left out the melody, the timber and the rhythm. Which better melody than the sung one? Which better rhythm than the beaten one? Which better timbre than the richness of percussions and electronic?

Et voilá, I have decided my instrumentation.

I did not compose peculiarly for this instrumentation in the beginning, but afterwards I shaped a lot all my compositions to it of course. Since there were very limited players, it was necessary to define all the texture possibilities with extreme attention and accuracy. Thus, even if I finally left a lot of space to improvisation, I took care a lot of the instrumentation and generally the partition of roles in the music.

If the reader is curios about this, I chose the percussion instead of the drums since they blend clearly much better with the acoustic guitar and the voice. Newly, this trio setting covers (in its essentiality) the parameters of timbers and rhythm (percussion), melody (voice) and harmony (guitar) even if all these roles are exchanged between the three of us. Sometimes the voice has a rhythmical role while the percussion has a melodic one and so on.

Michele Rabbia is one of the most asked percussionist into the European jazz scene. He has collaborations inter alia with Eivind Aarset, Andy Sheppard, Paolo Fresu, Marc Ducret, and many others.

Furthermore, the method gives different meanings to the material and links many styles from cultures far from each other. Thereby that can be found tracks of different styles and genres, all of them transfigured and merged into a hybrid constellation. I just reminds that the method is not only used to compose, but it also works as a way to get familiar with these new sounds and to phrase with quarter-tones; in addition, it opens up new possibilities for improvising within traditional structures of harmony. The reader now becoming listener will be able thus to recognize some quarter tone improvisation on different harmonic context, even on traditional harmonic choruses.

Each piece has its own characteristics and it follows a determined style of improvisation according to the harmonic rules that are unique in each piece, and which **never repeat.** I go now through piece by piece explaining the way the are constructed and explaining time by time all the compositional items I have constructed fitting the method into the compositional process.

6.1 Analysis of the pieces: exploring possibilities of the method

I here analyze the pieces I have chosen from the recording work.

I can not attach all the complete audio files from the recording session since they are possibly going to be released be a label and therefore they will be covered by copyright.

Furthermore, the audio files here presented are not the ones included into the recording; they are alternative takes not to be released.

Nevertheless some of them are attached as mp3. I have chosen the ones I believe they are the more representative of different techniques, for every pieces which is analyzed I present here the score, but again, not all the scores have the related complete audio file.

They are meant to be an example of artistic application of the method, thus, even if they represent an achieved artistic goal of the research, they are not fully integrated in it. However it is attached an audio file example for many of the scales here analyzed. These examples are instead completely belonging to the useful material of this research: it is useful to hear and therefore better understand its essence; when the professional recording could be rather considered as an example and /or inspiration.

Before to jump into every tune I must discuss the notation I have adopted in order to write quarter tones, so to clarify all the symbols that the reader will face in the next pages.

6.1.1 Legend regarding quarter-tones

There is not a common and universal accepted notation to represent quarter tones into write music. There are some historic notations in the context of Arabic classic music; and there is also an extended notation related to microtonal studies, that can be used for this goal.

As previously explained, I have used Lilypond to write the scores; consequently, the notation used to represent quarter tones must be choose from the ones supported by this tool. Lilypond gives two different modalities to represent accidents that can be positively used to represent quarter tone alteration: arabic tradition music and microtonal music accidents derived by The extended Helmholtz-Ellis Pitch notation. [2]

I consider more expressive the microtonal notation, so I have decide to use it the write the accident in the score.
Unfortunately, the cords into a sheet in Lilypond cannot be defined using the microtonal accident notation, but only using the arabic classic notation.
Consequently to our decision, in the proposed scores are used two different mode to represent quarter tones accident: the microtonal extension for notes and key signatures, and the arabic notation to define chords.

Here follows the legend which reports the symbols I have used to represent quarter tones in the different musical contexts, and must be keep in mind to correctly interpret the scores attached to the thesis.

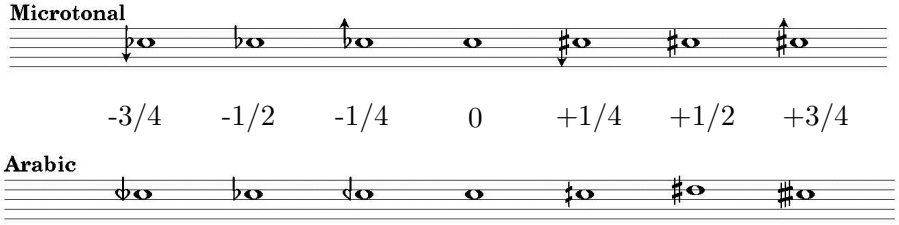


Figure 6.1: Accidents notations

6.1.2 Rituale n.1 Cerchio Terra and Rituale 2 - Triangolo Cielo

For every tune I report previously some of the most used scales into the performative and improvisational process, then I report the score it has been adopted to play the tune.

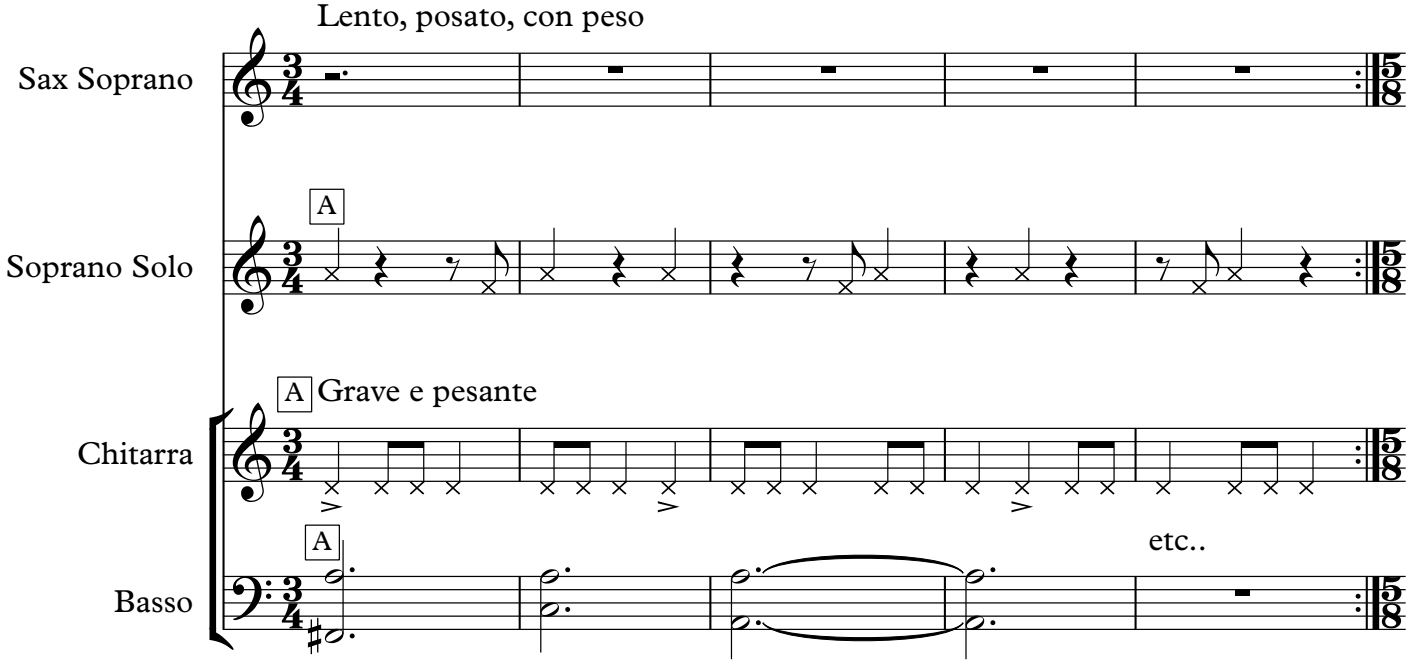
Reference scales:

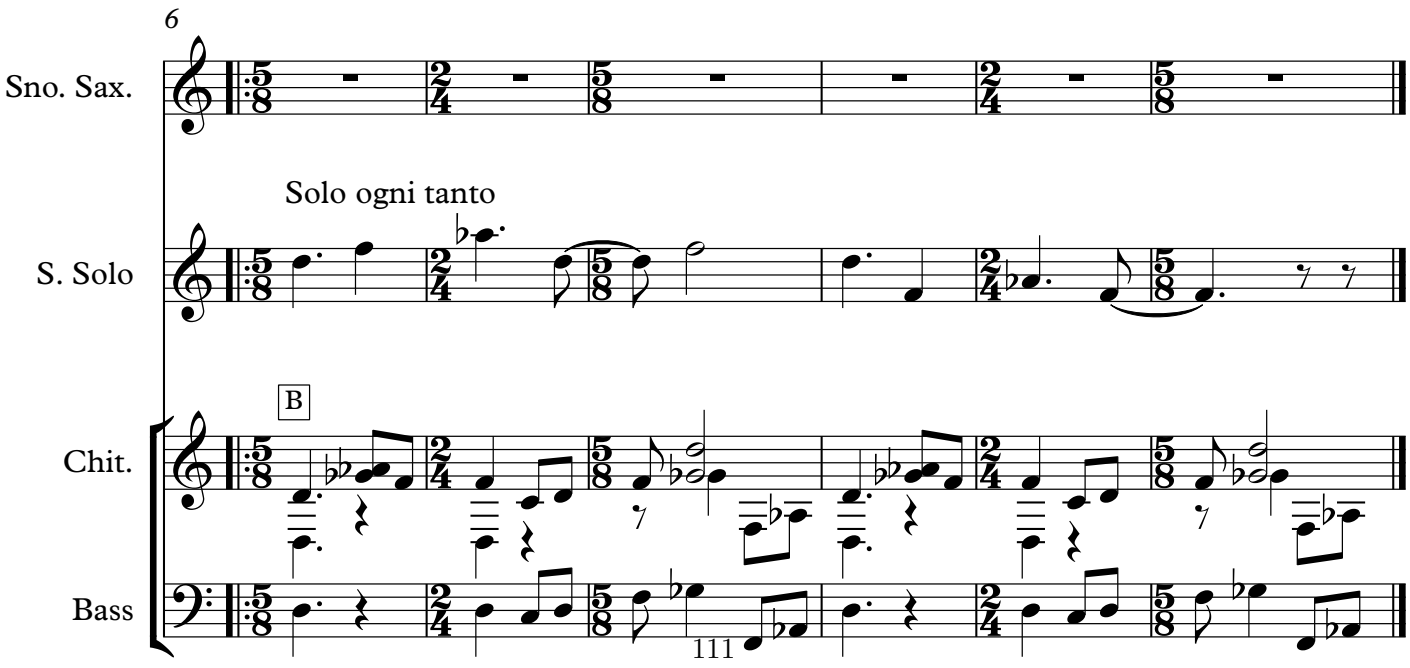
- {C ; D+ ; E♭ ; F♭+ ; F♯ ; A♭+ ; A ; B♭+ ; C} [3 + 9] absolute scale in C.
  (♫ audio 1.1)
- {C ; E ; F♯ ; G♯ ; A ; B ; C} [C lydian augmented]
  (♫ audio 2.1)
- {D; E♭+ ; F ; G♭+ ; A♭ ; B♭- ; B ; C♯- ; D} [3 + 9] absolute scale in D.
  (♫ audio 3.1)

[2] Marc Sabat and Wolfgang von Schweinitz. *The Extended Helmholtz-Ellis JI Pitch Notation microtonal accidentals*. [pdf], 2004. Available at: `http://www.newmusicbox.org/assets/72/HelmholtzEllisLegend.pdf`. [Accessed 5 Jun. 2017].

Rituale 1 - Cerchio

Enrico Degani





Here we face the first proposed composition which is actually also one of the first pieces I have composed with the method.

I always suggest the reader to hear the related audio file before or simultaneously the reading of the text and the score related to it. Only in this way it is possible to have a pragmatical idea of the topic we are facing: to listen the sound of this music is really important to understand the analysis we undertake here.

This first composition, is entirely built upon the symmetric octagonal scale coming from the geometric relation [3 + 9].

Thereby all the harmonic and melodic material comes out from the notes belonging to that scale. Nevertheless, even if in the composition I have used only this material, in the actual execution I also used the lydian augmented scale, as a balancing comment to the octagonal diminished scale.

Finally, they constitute the harmonic and melodic material of the tune: all the notes which are used here both to compose and improvise come from these scale and others improvised fragments.

However the harmony is not the distinctive point of this composition, rather its context of developing. The main idea of the piece is to give different rhythmic roles to each musician; to let the possibility to every musician to pursue a completely independent role in the play. Each and one is independent from the other, everyone keeps his own identity. It derives that musicians can both use the rhythmic and harmonic/melodic material of the tune and use different features while improvisation.

In other words: the goal is to obtain a choral effect while single individuality are expressing themselves independently.

I decided to adopt this organization in music after the reading of a book from Victor Grauer. [3] Grauer, in this book, describes the hypothetical primitive music through the analysis of the Bushmen and Pygmies music (he carries on an anthropological study he names as *anthropological archeology*). He looks at the structure of music events in those tribes and further on he analyzes them making a comparison to their social structure. Their society is based on the principle of the greatest individual freedom, where (differently from our contemporary society) individuals never fight to increase their positions into society: everyone contributes to the essential needs but there is not hierarchy and a pyramidal structure of society. Their music is thus structured the same way, everyone is free to enjoy and contribute to the music but at the same time single identities are conserved and there is not competition to shine over the others (there is no solo sections, every time it is a collective solo section).

In this kind of music, musicians are essential and necessary to create the music but at same time they are free to change their own role during the performance. This concept of music,

[3] Victor Grauer. *Musica dal profondo.* Le scienze - Codice Edizioni, 2015.

together with a sense of community, brings to a choral and collective execution.

In Grauer's book there is a lot of discussion upon the deep connection between social structure and culture. He is assuming that, since culture is product of a determinate community, culture is mirroring the way that a society is structured. Since music is of course part of the culture phenomena, it also mirrors the social structure of the community to which it belongs.

Finally, he is analyzing the peculiar characteristics (according the methodology proposed by the great anthropologist Alan Lomax and its *cantometrics*) of music relating its properties to some sociological issues.

Regarding the previously discussed music/society organization, since this organization of the form (which leaves lots of space to improvisation) the music material is very little. The mood of the piece concerns the rhythms figures, the timbre aspects rather than the melodic and harmonic devices.

Made this premise, it is now quite consequent to affirm that I did not developed to much tools regarding quarter tones harmony and melody: this piece adopts probably the most simple and direct approach to the subject. I just decided a scale and I have used that as a modal/texture context for improvisation.

There are therefore some fixed riffs which anyway are useful only to give hints to the performers which are all the time inventing new riffs to be juxtaposed one over the other one.

Harmonically we could define this as an *hétérophonie*, since it can randomly occur some kind of cadences (rather some axis and sound contexts) and movement into the scale but generally all the combinations of notes into the frame of the mode are accepted.

In other words, vertical moments are not structured and controlled, they just occur as the juxtaposition of multiple independent lines.

Furthermore, in my personal idea of music and thanks to the aesthetic idea that I have been developing by playing this tune, I personally think that the eight-tones octagonal scale recalls a fake primitive music (I say fake since of course we do not have so many sources regarding it).

Concerning the language I adopted in my improvisations here, I use the leading-tones (typical in bop language) with quarter-tones intervals instead of half-tones. Therefore the improvisation relies a lot on the jazz language: rhythm and phraseology are very important and chromatic scale is really useful the shape the rhythm of the phrasing.

Furthermore, since this composition is set into a twenty-four notes context, approaching notes might here be quarter tones; the chromatic scale might became the ultra-chromatic one (as named by me already before in the text).

I want just to stress again that in this piece I use a perfect symmetry scale (the octago-

nal scale), thus this tune is an example of the usage of those special items (they are not diatonic and so they do not express tonal areas, therefore they are very flexible). Since there is not degrees belonging to tonal areas, there is no movement order rules, no cadences. I have then used clusters and lines that move freely among the eight-tone scale; due to this property it is possible to move around without any limitation.

Going into details, the piece is set in two sections: the first one in C, the second one a tone above, in D. While the first part is more free; this second section is characterized by an obstinate riff in 7/4 which is kept ahead by the guitar. This small form construction contributes go give a better overall shape to the piece.

Concluding the analysis, I want to spend some words regarding the importance of the extra-musical inspiration in this tune. Since there is so little composed music and just a few harmonic material, it is very important to set a precise *image* to define the identity of the tune. Thereby when I had to record it I put a lot of focus on the effort to make this particular ritual (I have composed four of them, all with the same concept) sound as it is named: *Terra*.

Terra means earth in Italian, thereby the mood of this ritual is defined by the heavy and dragging feel of the execution. Thus the idea was to create the sensation of being earth grounded, being heavy and surrounded by an ancestral nature.

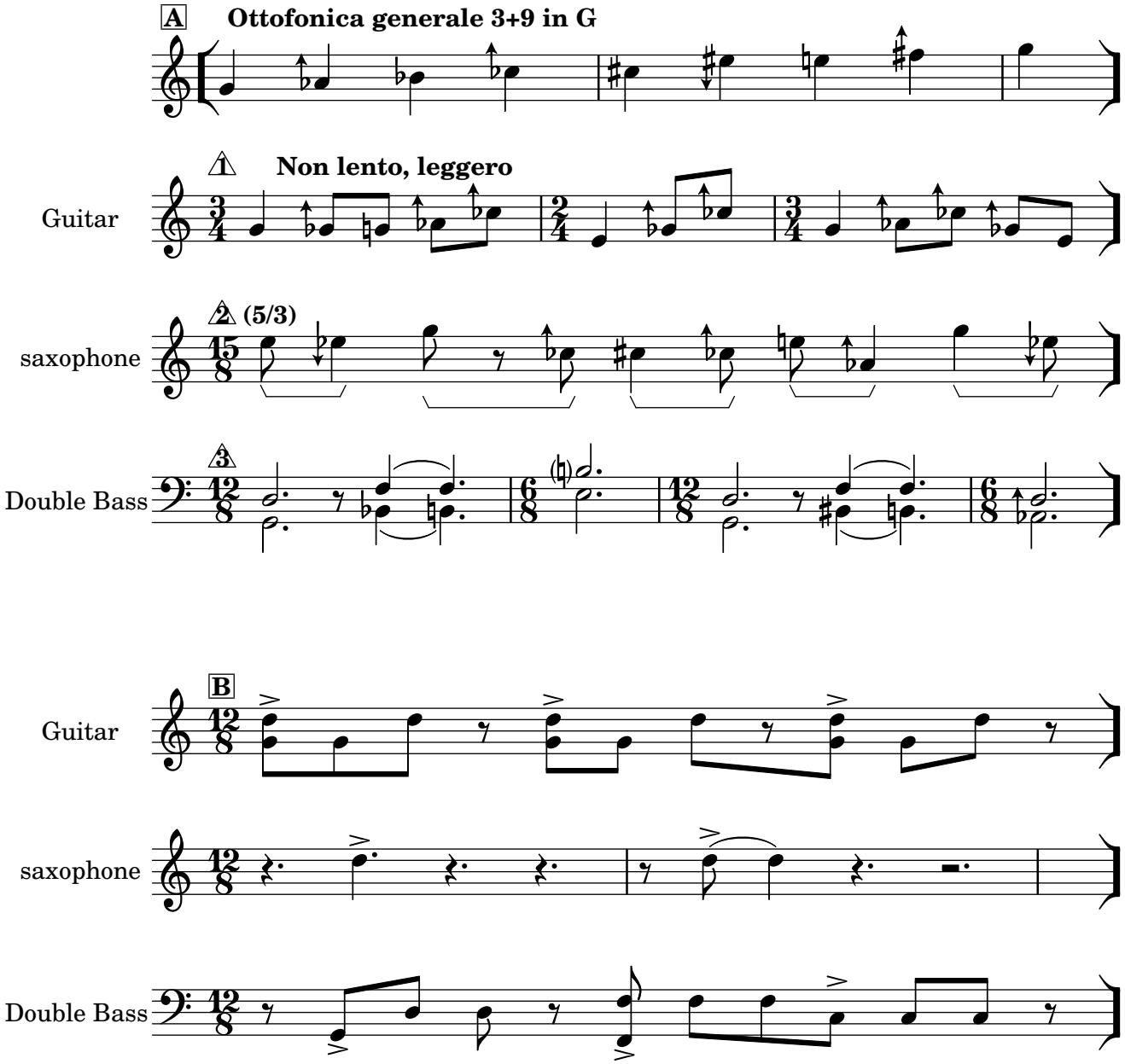
I hope that the reader could have enjoyed the words meant to correlate the audio file, nevertheless there was not so much to talk about on the harmonic issues regarding this piece. There is much more to say about the following proposed tune, in which I have constructed on purpose some compositional tools which make use of the method.

Nevertheless, before to go on I want to share with the reader also the *Rituale 2: cielo* score; so that she or he can observe that they share the same constitutive structure into a different expressive context.

Rituale 2 (cielo)

Triangolo

Enrico Degani



Notes to the execution

A B A ... A : free develop of this pattern, but ending must be on A

About voice:

Voce sempre libera e/o doppia su una delle parti.
Funzione ritmica, suoni percussivi ed onomatopeici.
Cantare solamente le note intonate.

Music engraving by LilyPond 2.18.2—www.lilypond.org

The Rituale n.2 is based on a 8/4 riff proposed by the guitar, which is set as polyrhythm on the top of a 12/8 pulse.

As it can be observed, the form of the tune is the same as the one I adopted in Rituale: n.1.

Nevertheless, the mood here is suggesting opposite feelings compared to the other one. Here the key word is "sky", and then lightness, fast and fresh melodies who are running one to the other one. The music material is the same for all these two *Rituale* pieces: the octagonal symmetric scale.

Again, the difference is generated by the timbre and rhythms features, along with visual inspirations.

In the recording, one ritual is for the earth, one for the water, one for the sky.

6.1.3 Quarta dimensione (In between)

For every tune I report previously some of the most used scales into the performative and improvisational process, then I report the score it has been adopted to play the tune.

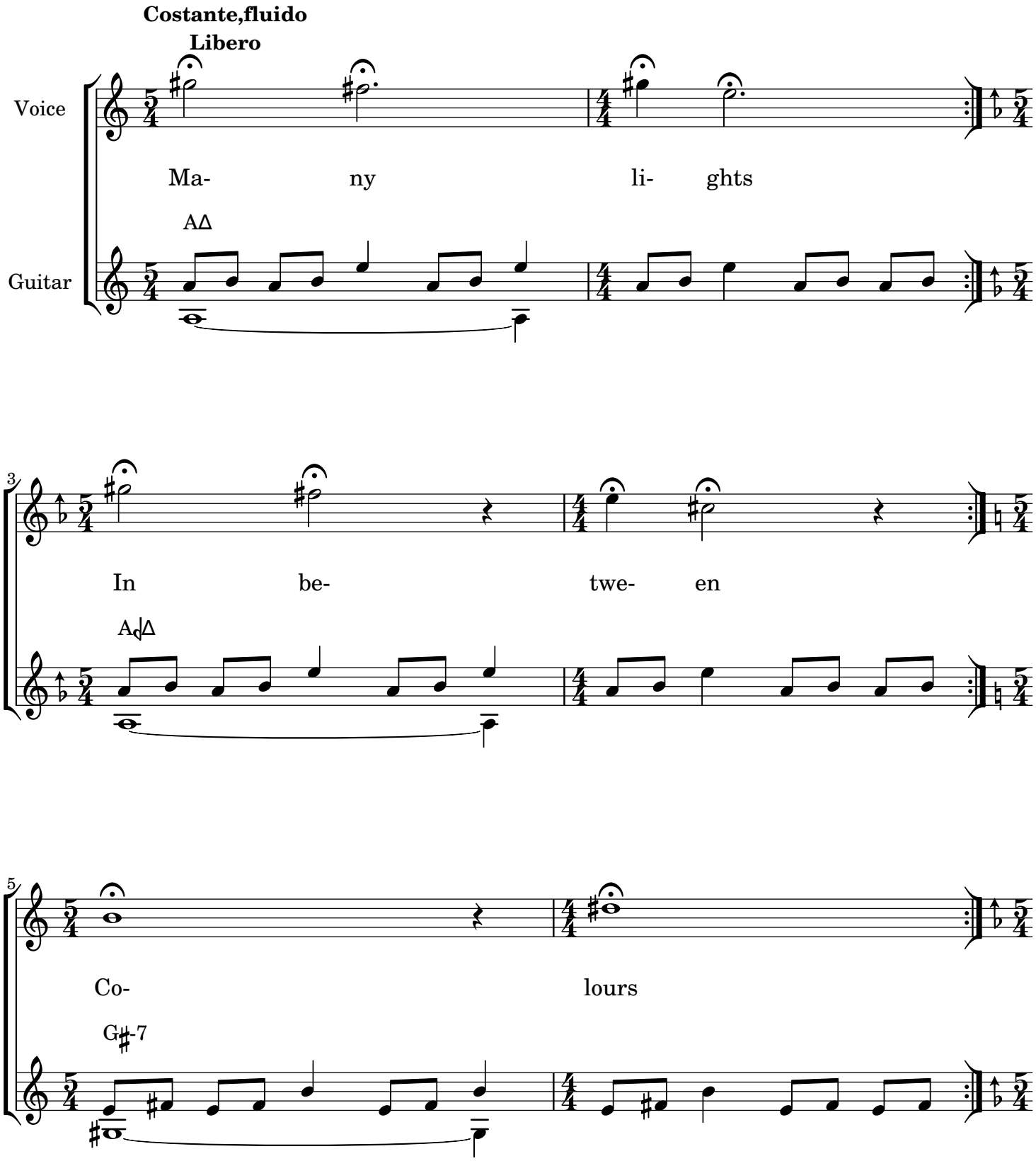
Reference scales:

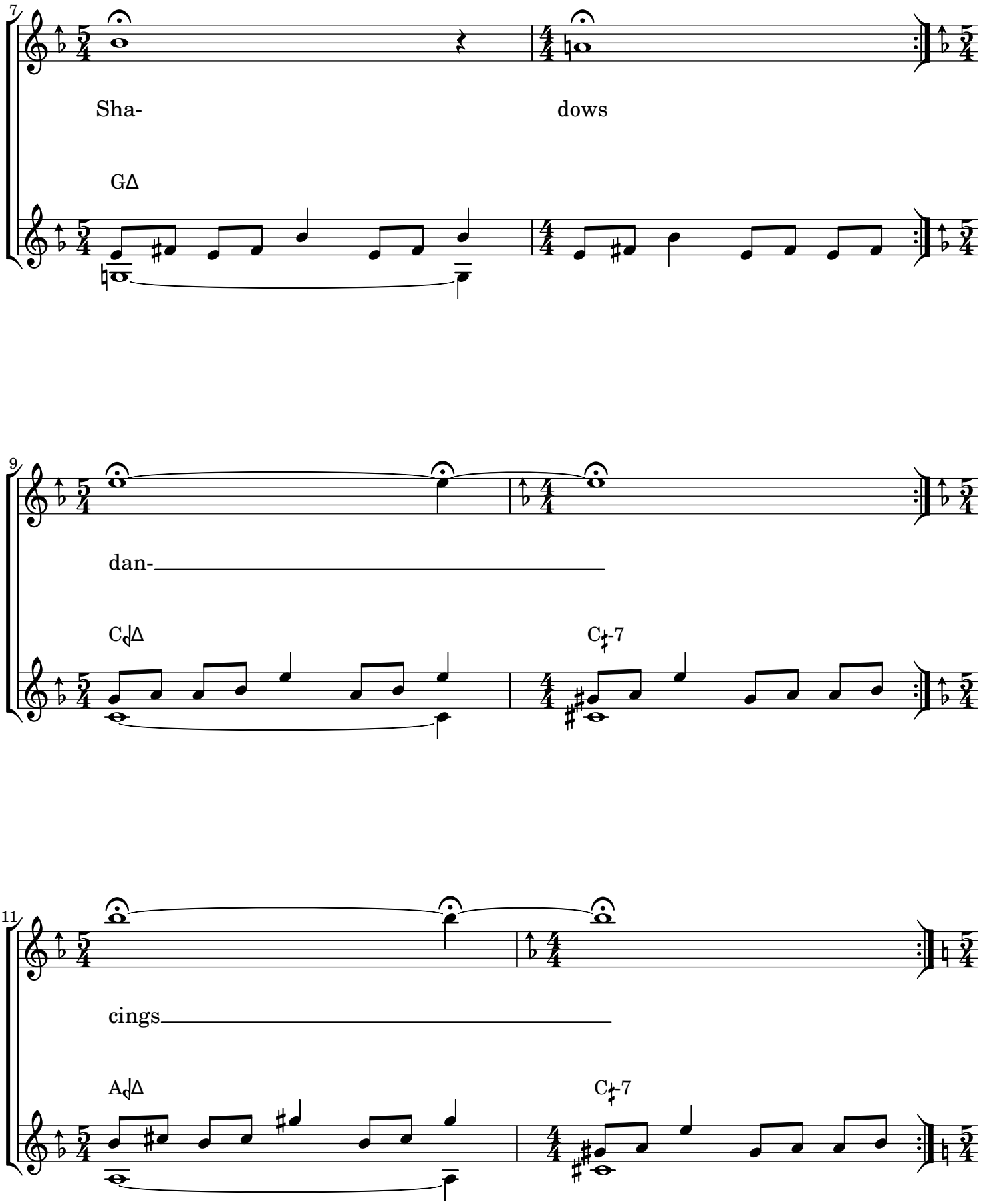
- {A♭+ ; B♭+ ; C♯- ; D♭+ ; E♭+ ; F♯- ; G♯- ; A} [A♭+ major scale].
  (♫ audio 1.2)
- {A ; B♭+ ; C♯ ; D♯ ; E ; F♯- ; G♯-} [A Lydian generative from quarter tone pentatonic]
  (♫ audio 2.2)
- {A; B ; C♯ ; D♯- ; E♯- ; F♯ ; G♯- ; A} [A Lydian augmented generative from quarter tone pentatonic]
  (♫ audio 3.2)

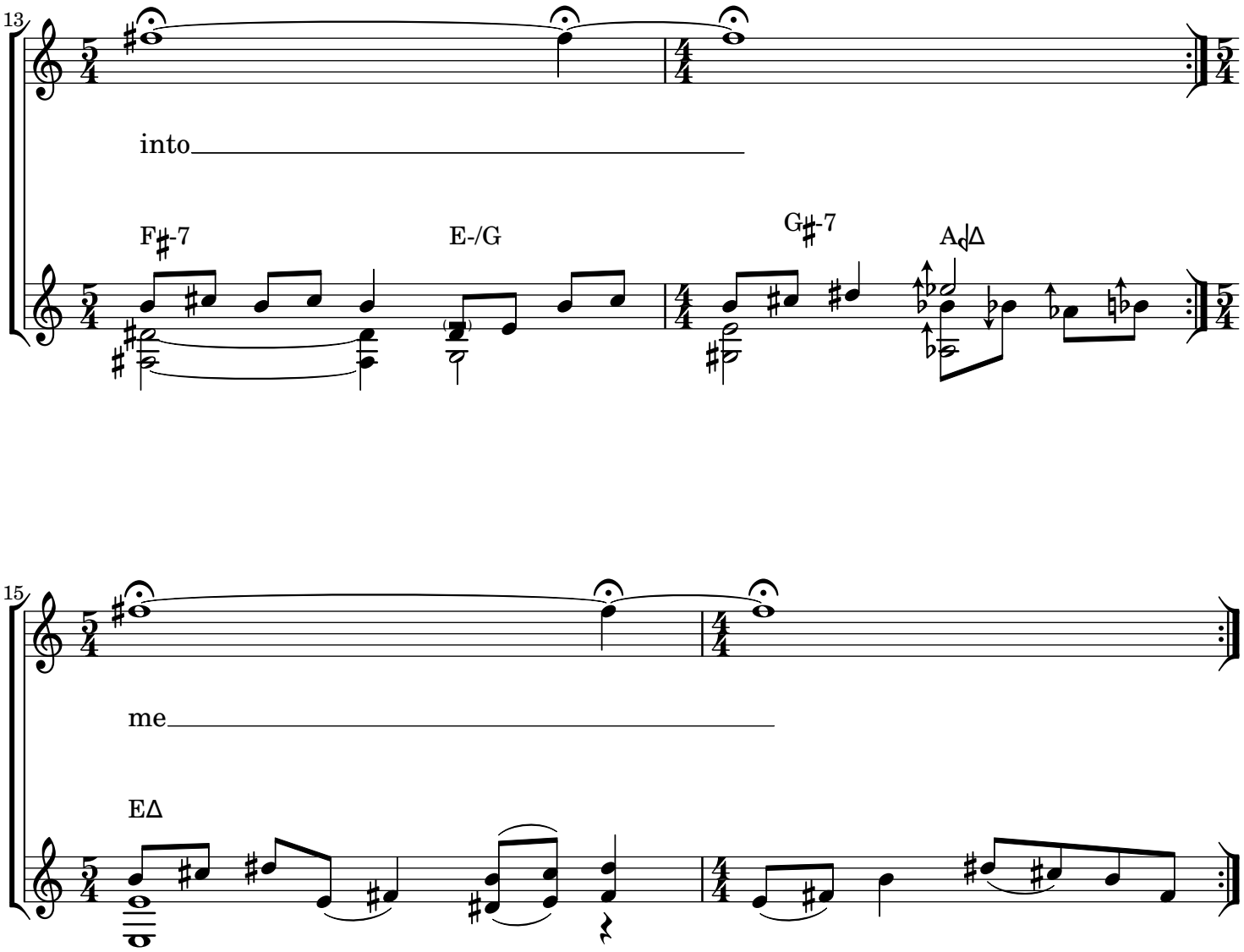
Quarta dimensione

in between

Enrico Degani




7
Sha-
dows
GΔ
9
dan-
C♭Δ
C♯-7
11
cings
A♭Δ
C♯-7



As said in the beginning of the chapter, in every proposed tune I use a particular technique I do not use in the other tunes. Therefore this tune represents a new compositional item in which I go through now explaining it.

One of the hardest issue to resolve approaching the quarter tones was the chords; the superimposition of spare intervals (1 Qt; 3 Qt.. and so on) generates pretty harsh cluster difficult to be digested by the listener into a tonal derived context.

Thus, even if I made easily use of all the scale got from the method, I had more problems regarding chords in harmony. As the reader might think, I have developed few strategies to bypass the problem. One of those is the one I have used to compose this tune.

In this tune all the harmony is constituted by *normal tuned chords*, there are not spare intervals into a vertical harmonic moment. In other words, all the vertically harmony (See 3.4.1) is traditional: it is about chords common in tonal music where quarter-tones never overlap.

However these chords are not always constructed starting from a note which can be found in twelve notes system. This means that all the chords are constructed with pair intervals (so they avoid quarter tones distances in between the chord) but they can be sometimes starting from a quarter note position. For example we will find an Amaj7 (A,C♯,E,G♯) and an A♭+ maj7 (A♭+,C♯-,E♭+,G♯-).

This way we introduce quarter tones and quarter tone distanced systems relation. We connect tonalities distanced by quarter tones.

This implies that all the harmonic and even melodic material has been dealt with a traditional approach into the tonalities. When for instance there will appear an Amaj7, the melodic lines, the chords and also the related improvisation, will belong to a traditional approach in the tonality of A major. Nevertheless the same will be in A♭+ major.

Therefore there will occur a relation determined by the shifting of traditional tonalities in places never explored before. We will have *quarter tone modulations.*

In other words, here I use tonal functions which can be found in twelve tone system but I shift them to points which can be reached only into a twenty-four sounds system. At the same time when I improvise I use tonal functions that can be applied according the chord they deal with, so I sometimes use quarter tones scales on the top of those chords.

Clearly, of course I use the A♭ major scale on the top of the A♭+ major section; however I also use some quarter tone diatonic generated scales to better connect the modulation.

It is clear that, the extended pentatonic become here very useful, since they allow to superimpose quarter tone lines on the top of a pair intervals constituted chord. Thus we will obtain some *pivot notes* that can be meaningful for an half-steps derived chord and a quarter-tone derived one.

For example let us consider this couple scale/chord:

Amaj7 - {A, B, C♯, D♯+, E♯+, F♯, G♯-}

and:

A♭+ maj7 - {A♭+, B♭+, C♯-, D♭+, E♭+, F♯-, G♯-}

We can observe that there is common notes such as G♯- and D♯+.

Therefore the usage on quarter tone scales obviously better connect quarter tones distanced tonalities.

The listener will therefore hear some weird modulations sometimes helped by connecting scales. In the proposed recorded version by the way, the quarter tone changes appear only as ending of the tune. Previously there is a long section in E major (or A lydian), and then the modulation appear as a *step through the mirror* into a intermediate dimension never explored before.

I decided to place a static introduction before the modulation in order to amplify the effect of them: things are much stronger when unexpected.

The title of the tune (In between, quarta dimensione) comes out from the *mirror* (winking Lewis Carrol's *Alice*) we can already find in the first bars of the score. In the beginning it starts with this progression: Amaj7, A♯-maj7,A♭maj7. Thus it slides from A to A♭ passing by the quarter tone step modulation. This gives the illusion to be in between the half-step, to explore a fourth dimension.

Further on in the composition, modulations between chords alternate quarter tone steps to half steps. This gives the illusion to our ear to always listen to an half step modulation when it is sometimes a quarter tones one.

This technique, far from any traditional music I have heard, brings back to experimentations done in Europe during the XX century (Alois Hába, Charles Ives). However it differs from them since it maintains a jazz style which implies improvisation. This double feature of the composition gives a special mood to this piece.

As said before, this piece moves slowly among chords giving the impression to modulate to another tonal system every time a new chords appears.

In other words this chord progression is not to be intended as a tonal progression, rather as a modal series of harmonies. There is not a scale which relates more chords, every chords has to be intended as tonality itself. So much so that on the chords the harmony shifts towards the degrees of its tonality ambit, claiming this principle such that every written chord on the score represent a tonal ambit rather than a tonal function.

In order to give the feeling of quarter tone modulation, I preferred to mainly play traditional tonal functions (as the major scale) on each chords, so that the listeners has the perception to a conventional tonal music which modulates to never experienced tonalities.

I pose again the issue regarding the *pivot* scales here following. As said before, the diatonic scales with quarter tones are the link every time there is a modulation from a tonal center to another one distanced by a quarter-tone. In order to have a smoother path between tonalities when the quarter tone distance appears I have used scales which contains quarter tones so there were some common notes in between the two tonalities.

For example let us consider the modulation from A major to A♯- major. I can use on A this quarter tone diatonic scale:

{A, B-, C♯, D♯, E, F♯-, G♯-.} R, 2M-, 3M, 4, 5, 6M-, 7M-

If we consider those two scales in A♯-major:

{A♯-major: A♭+,B♭+,C♯-,Db+,Eb+;F♯-,G♯-}
A♯-major quarter: {A♭+; B♭+, C♯-, D♭; E♭+; F♯-;G♯-}

We can see that F♯-, G♯- and C♯- are common notes. We have written here another example of *pivot* scale just to show to the reader the multiple possibilities given by the system to connect harmonies with useful scales. There would be much more solutions to be discussed, but for sake of simplicity we limit ourselves to two examples.

The curious reader might check in the addendum the list of scales and try to make hypothesis regarding the possible *pivot* scales in various contexts.

Concluding with a practical consideration, this is a very difficult procedure which, however, leads to a new extended concept of tonality and modulation within the twenty-four notes.

6.1.4 Under the old tree and Dust Dance

For every tune I report previously some of the most used scales into the performative and improvisational process, then I report the score it has been adopted to play the tune.

Reference scales:

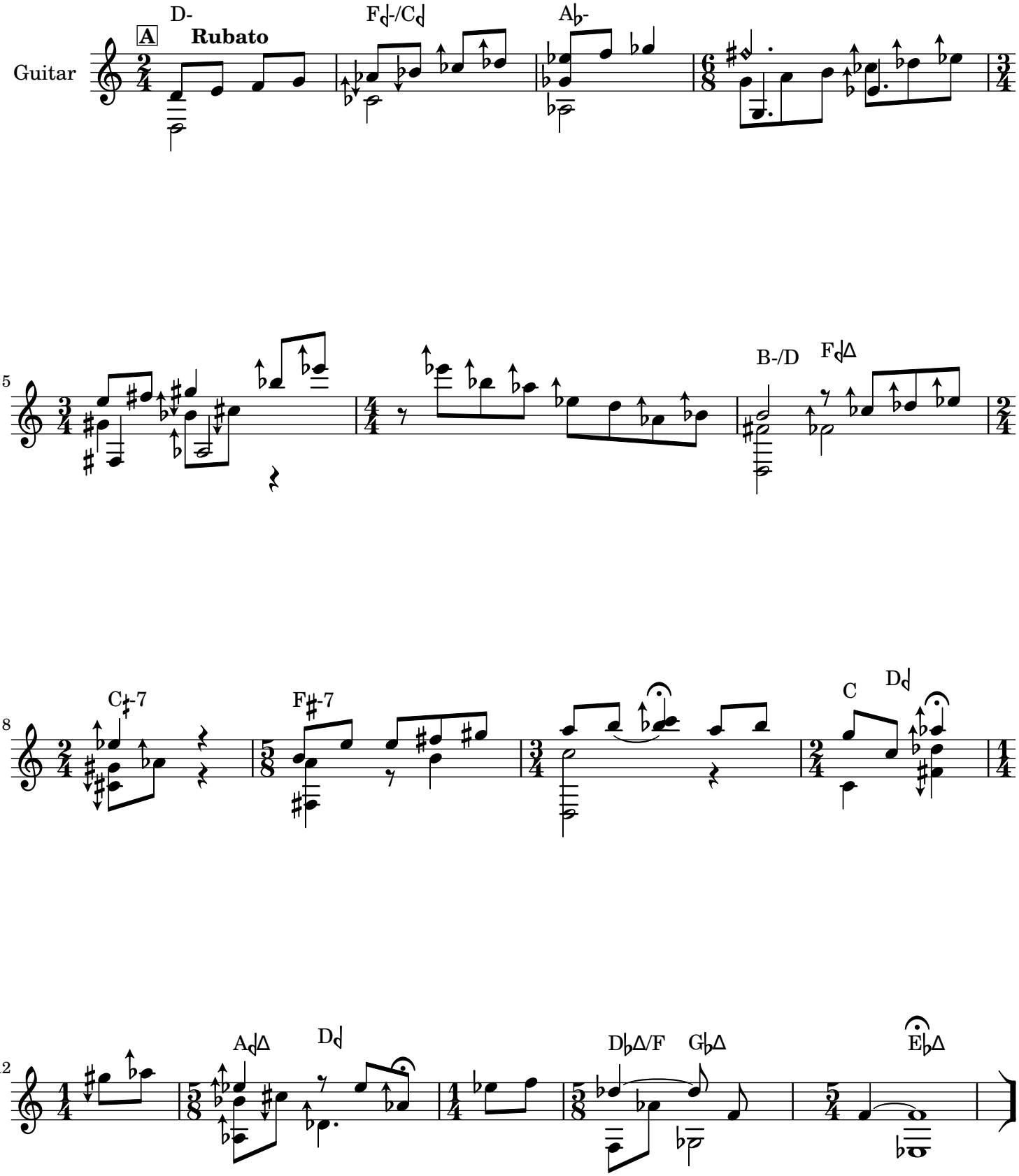
- {E ; F♯ ; G- ; G♯- ; A♯ ; B ; C♯ ; D♭+ ; E} [5 + 7] General scale in E.
  (♫ audio 1.3)

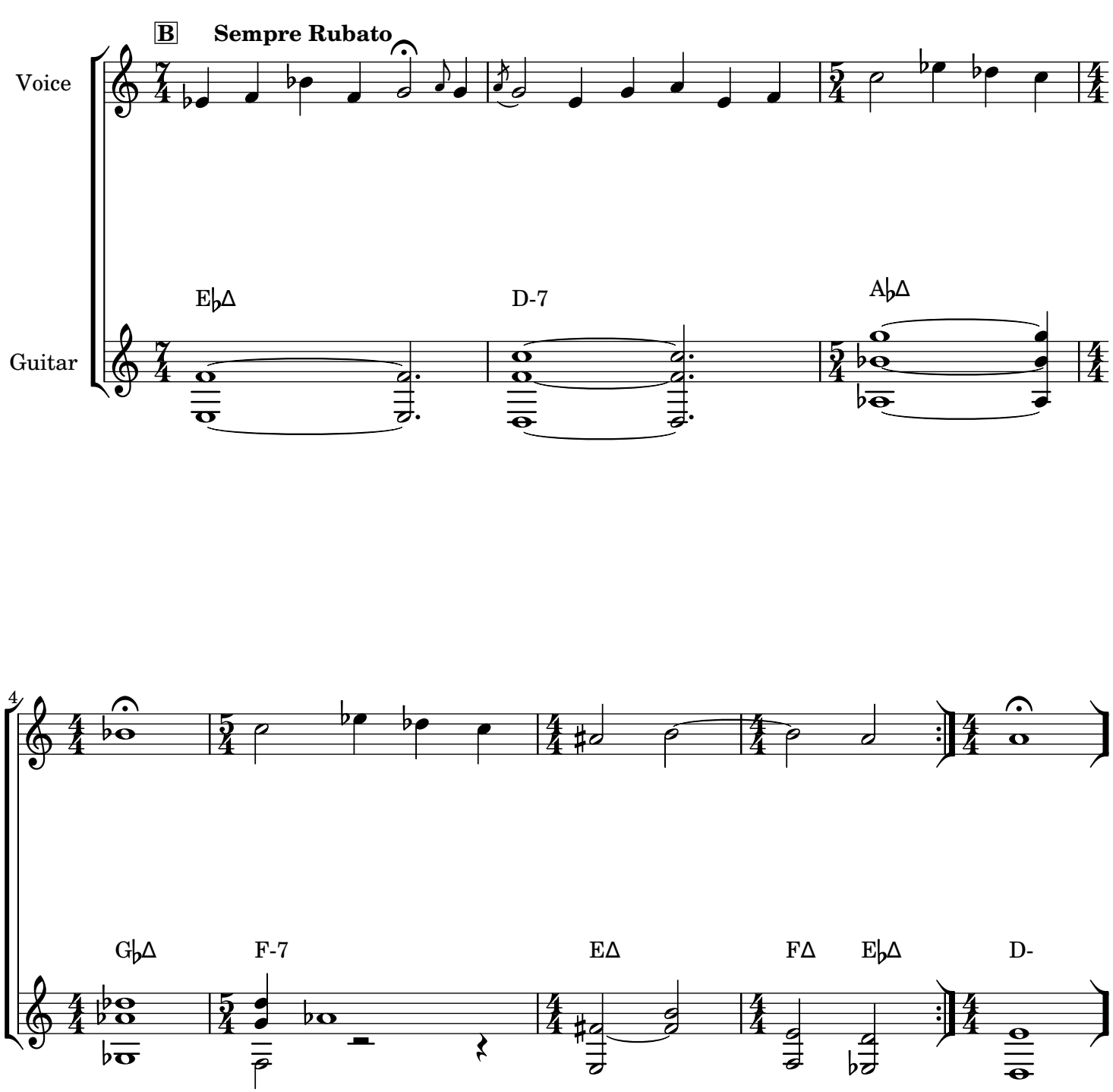
- {A♭+ ; B♭ ; C♭+ ; D♭+ ; E♭+ ; F ; G} [A♭+ Minor melodic generative from quarter tone pentatonic]
  (♫ audio 2.3)

- {D; E ; F- ; G ; A ; B♭+ ; C ; D} [D Minor Dorian generative from quarter tone pentatonic]
  (♫ audio 3.3)

Under the old tree

Enrico Degani





Music engraving by LilyPond 2.18.2—www.lilypond.org

Dust Dance

Enrico Degani

Vivace. Ritmico

Voice

In dust

Guitar

A

Asus

E♭Δ

3

we dance fast

C♯-7

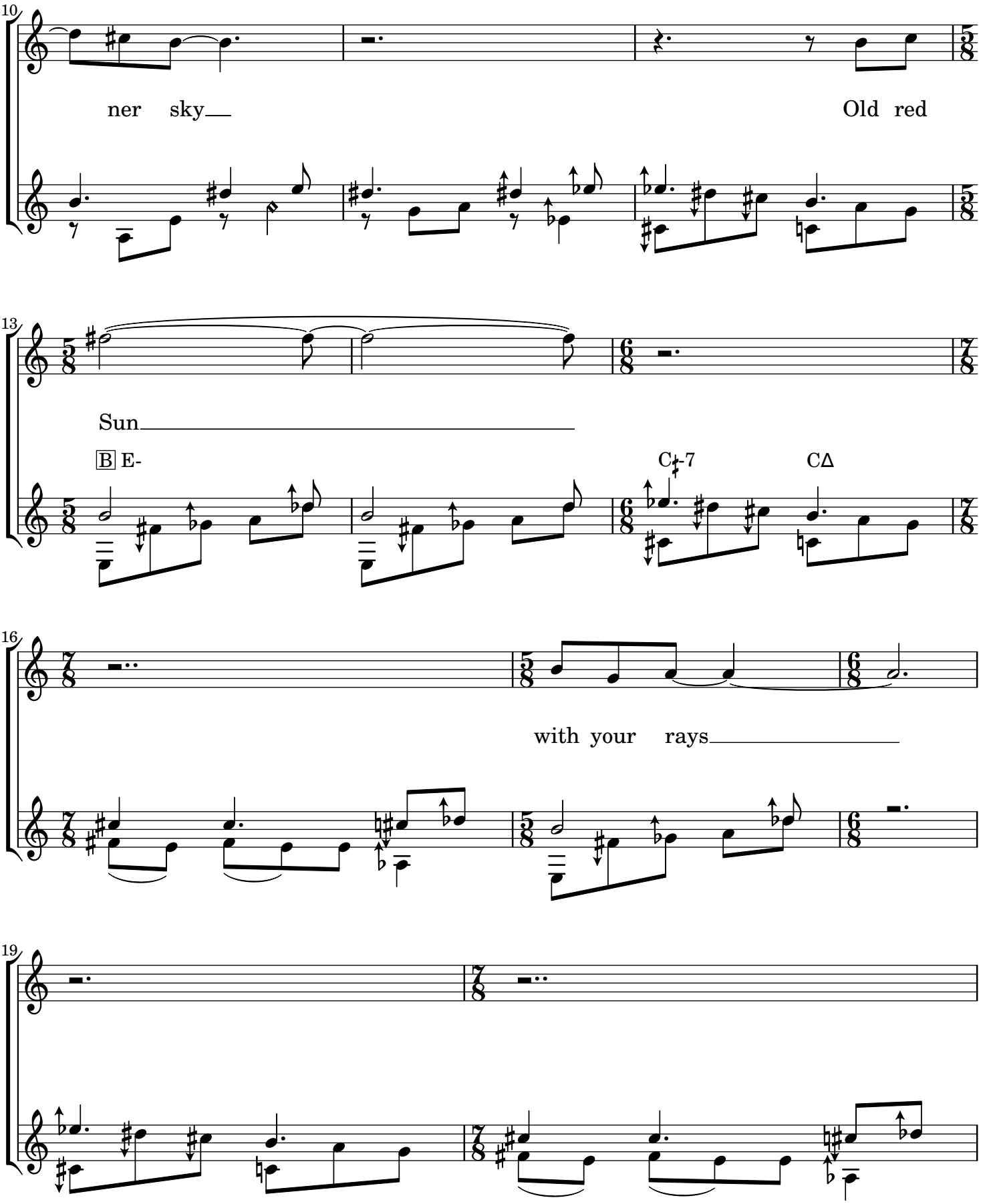
CΔ

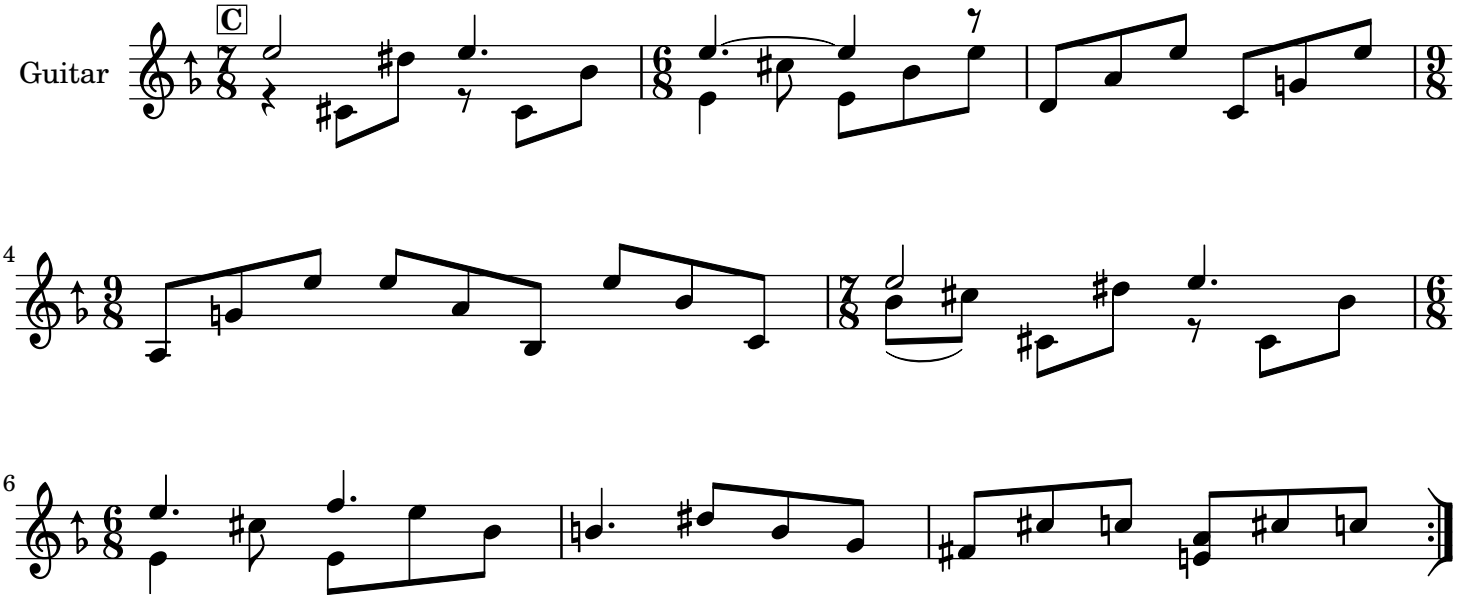
5

And O- ur

7

feet black 'n warm touch our in-


10
ner sky
Old red
13
Sun
B E-
C♯-7
CΔ
16
with your rays
19



Music engraving by LilyPond 2.18.2—www.lilypond.org

In these two compositions we face another compositional item created through the method: it is created a continuous movement through chords far off a quarter tones, which generate a new concept of tonality where tonal grades include quarter tones.

In these pieces, same as in *Quarta dimensione - In between* all chords are made up of pair intervals. Differently from there, chords here move quickly through the harmony using quarter-tones distances in a very various setting.

If previously we mainly had a connection between different tonalities, here we have a connection between chords. This means that if previously we could have found a C♯-7 on the top of an Amajor7 section, here every chord sign has a precise meaning that has to e observed. Furthermore we will not find only major chords, *au contraire* here they will appear various chords such as minor, dominant seven and so on.

This clearly implies a different sonority, less a modulation, more a continues path in which the ear shifts among the changes approach a full determined twenty-four notes ambient.

Thus, these compositions could be considered the most evolved ambit of the application of the method. Here both connections between chords, both scales with quarter tones are fully integrated into a complete system.

As the reader might suppose, these tunes are the most difficult and play and especially to improvise on (since they require a really deep knowledge of the quarter tones scales and their functionalities depending on the chords). Thereby the performance does not make use of the full possibilities of the method in this ambit, it will hopefully come with time.

In these compositions, chords can be gathered by scale and generally I adopted a simple voice movement expedient. Here the connection between chords is possible thanks to a descendant slide of one or more voices with a quarter-tone step.

All the time a line goes down of a quarter tone, the result is pretty well accepted by the ear, things which does not happen when we go up of a quarter tone.

Since of course all the half steps movements are allowed, the link between chords has to respect two rules basically:

- all the chords have to contain only pair steps (no quarter tone clusters in the chords)
- the movements generally allowed are: half step up, half step down, quarter tone down.

Of course chords can be put in relation also following the rules of tonal cadences both for a *half-step tonality* both for a *quarter-tone tonality.*

This means that if we adopt the just exposed rule to connect chords into a sliding movement, we will move always from tonalities to other tonalities (even if for very short time). Thus, whenever we will set a tonality, there we will connect chords as they would have been connected according to the traditional tonal harmony rules (even if we land in E or in C♯- major).

I take bar 5, *Under the old tree* as an example of the sliding process:
I move from the chord F♯/G♯/F♯(E) = F♯min7 to the chord A♭+/B♭+/G♯- = A♭+maj7.

The upper voice moves from F♯ to G♯- making a 3 quarter-tones step. The bass voice makes a disjoint jump, since it is defined by the harmony of the two chords. The medium voice also moves according the harmony.

This practice leads to a more counterpoint oriented way of making music. Those chords are linked without a tonal common ground. This drives to a more independent role of the parts, allowing the frequent change of tonal center with elegance.

If the reader might ask why do I always refuse chords which are made up of spare intervals, I answer saying that when there is a strong tonal sense in the composition it is not good to place a chord made up of odd intervals (which contains quarter-tones in its structure). This is due to the music ear ability to recognize the similarity with the conventional tonal system. It determines a perception of those chords as "out of tune", as wrong.

Au contraire, if we listen to the "rituals" pieces (which I talked about previously) the superimpositions of odd intervals are totally accepted by the ear since they are not inscribed into a tonal context (music is rather constructed upon a symmetric scale).

However, in arts rules are never that compulsory, thus in *Dust dance* odd intervals chords rarely appears. This seems to be a contradiction with the rule soon announced, but again, as we all know music practice accept exceptions. Nevertheless these chords sound still weird but of course when we compose we can use every item we think it is useful to the music.

Concerning the improvisation, here it will appear both conventional scales used in different chord/tonal center, both quarter-tones diatonic scales into a constant interchange (as it was in *Quarta dimensione In between*). I just want to remark again that *These are the most difficult composition to be played since they require a complete and deep knowledge of all the scales possibilities in different keys (including the quarter tone rooted ones).*

In *Under the old tree* the B section is constituted by a normal harmony chorus where the voice improvise a solo form, in *Dust dance* there is always a coexistence of half steps and quarter tones, even in the impro section.

I now move on to the last ambit of composition.

6.1.5 An old tale and rubino

For every tune I report previously some of the most used scales into the performative and improvisational process, then I report the score it has been adopted to play the tune.

Reference scales:

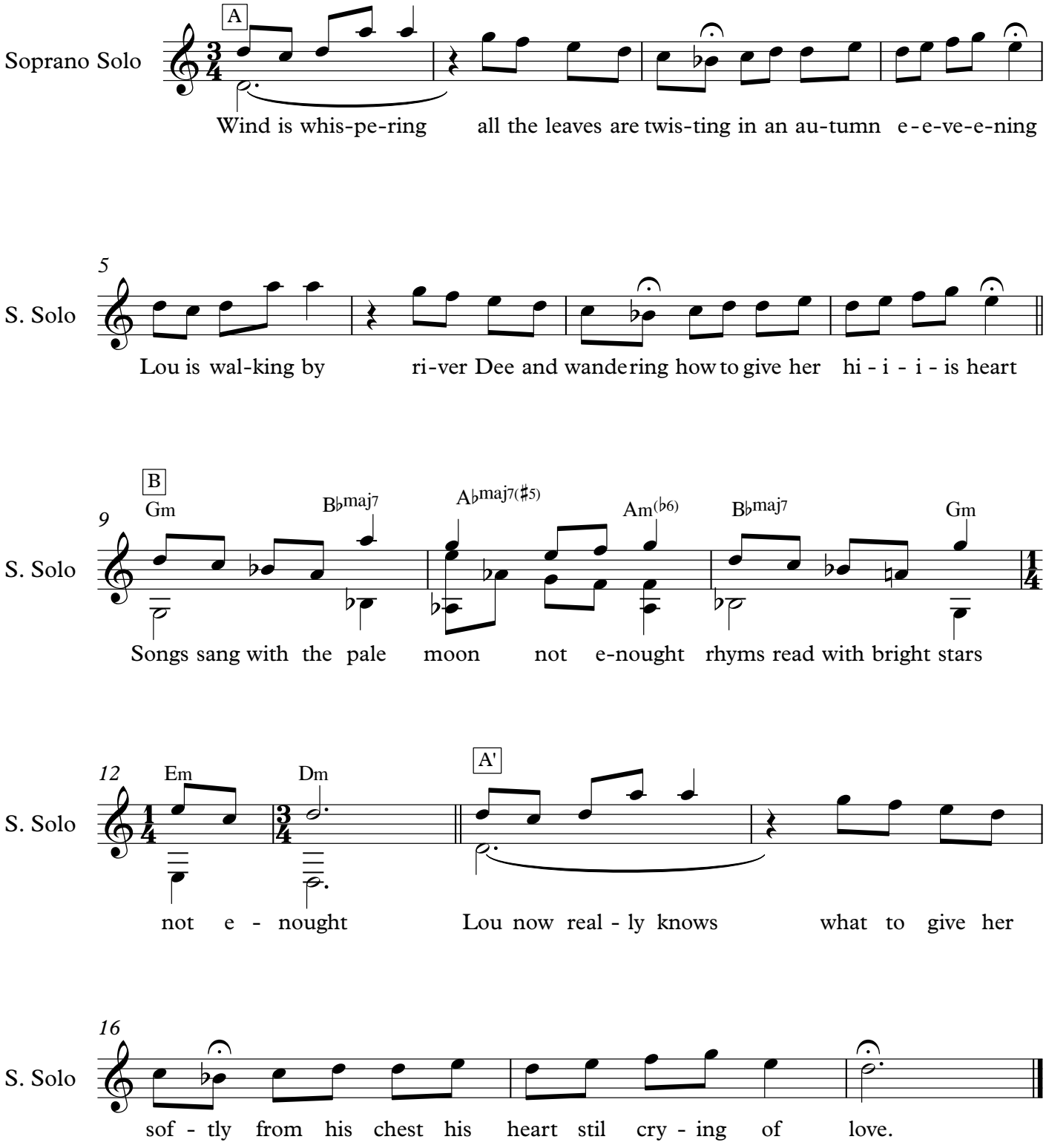
- {D ; F- ; G♭+ ; A ; C♭+ ; D} [Quarter tone pentatonic]

(♫ audio 1.4)

- {E ; F♯- ; G ; A ; B ; C♭+ ; D♯ ; E} [E Minor harmonic generative from quarter tone pentatonic]
  (♫ audio 2.4)

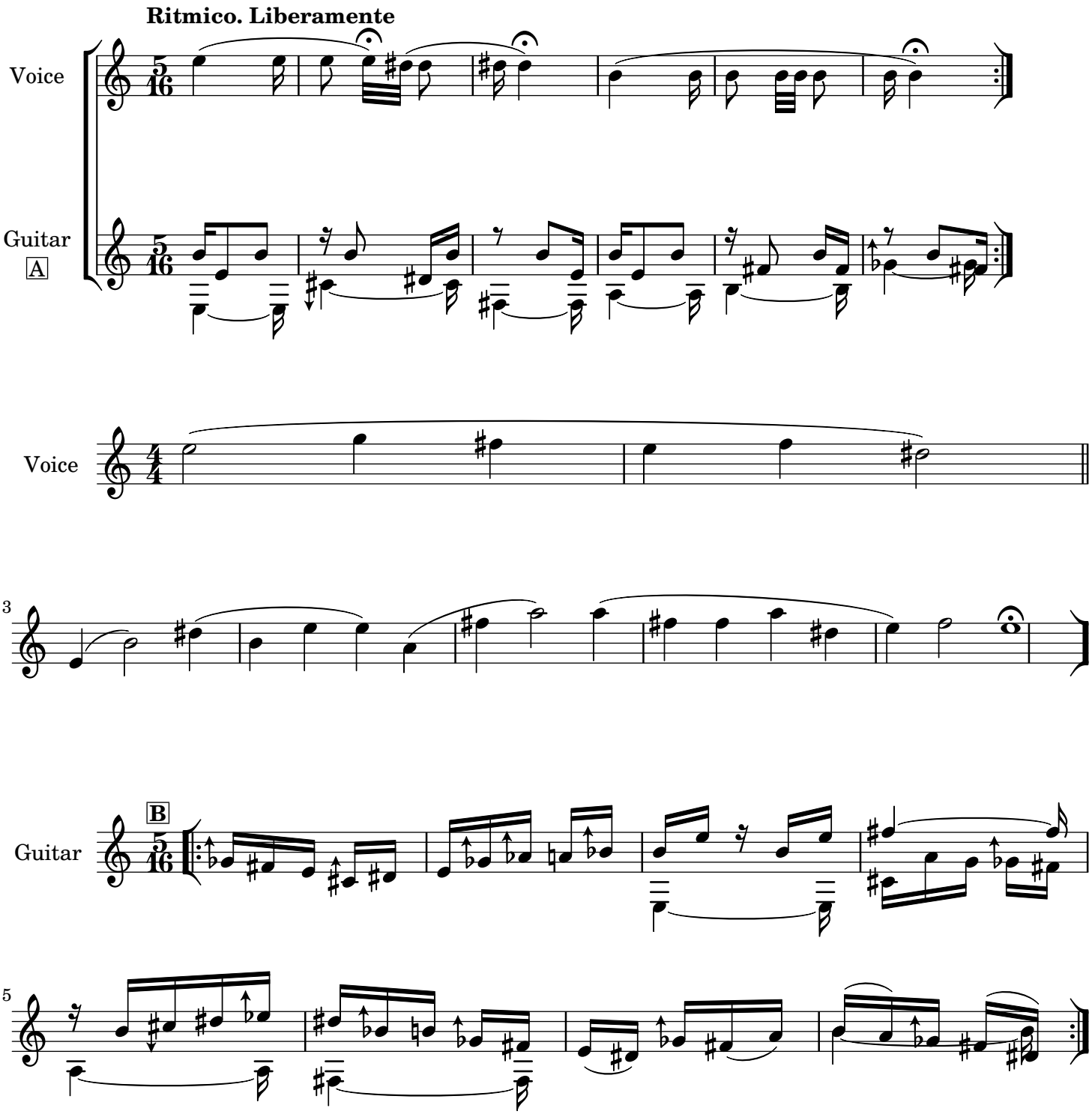
An old tale

Enrico Degani



Rubino

Enrico Degani



Music engraving by LilyPond 2.18.2—www.lilypond.org

These two compositions, which differs a lot from expressive and rhythmic features, share the same idea in the usage of the harmonic and melodic material.

These two composition are probably simpler than the previous one since they adopt a modal conception, which is pretty close to the Arabic music (and thus probably easier to approach since *already existing*).

In these compositions the harmony is mainly a pedal above which they are being played different possible scales.

Moreover, in these compositions I mostly used scales derived from pentatonic. I here use mainly all the possible diatonic developments of pentatonic scales we find in triangles schemas. Thus, I here consider the list of diatonic scales grouped by diatonic context.

I just remind that, as said in the methodology (See 5.5) the pentatonic derived from triangles are developed and grouped by diatonic characteristics (minor melodic, minor harmonic and major).

From a couple of values [2a + b] there will be pentatonic which will be developed in different diatonic context. Therefore there is a double list of diatonic scales coming from triangles: scales in different diatonic context can belong to the same triangle figure. Therefore in these compositions, I had to re-schedule them by diatonic context in order to use them here in these modal context.

As stated before, the modal conception drives us to the Arabic world. These scales reminds also the Arabic Maquam system and the harmony behind them is always a modal pedal, as in the Arabic music. Both tunes are built on a pedal (D for *An old tale* and E for Rubino) and different minor and altered scales are played on the top of them. However there is then a free use of various minor and altered scales on the same pedal, which is more a jazz feature then an arabic one where the scale mode remains the same for the whole tune.

Since the tonality does not appear here it is absolutely allowed to have all kind of chords and vertical simultaneity freely made up of quarter tones and half steps. Both in improvisation and in composition all the possible combinations of keys in the scale are admitted as long as the pedal note remain fixed and determines gravity.

If *An old tale* is a really simple song, later on adorned with quarter tones, *Rubino* has already quarter tones in its written score and it is built with a complex polyrhythm of five against three. I would like to say just very briefly this polyrhythm I learned it from my friend and colleague Andrea Ayace Ayassot. [4]

[4]We are releasing a record of his own project and compositions, to examine in depth this technique, when all those rhythms are used. To have some suggestion about the musical universe of Ayassot, please, refer as example to the online video:
Andrea Ayace Ayassot. *Incantabile* Available at: `https://www.youtube.com/watch?v=ODBuh-DSHgY` [Accessed 3 Jan. 2016].

In conclusion, these compositions are the ones in which I had the occasion to try out more scales without the risk to occur in harmonic problems. Thereby they might be the more interesting regarding the improvisation, while regarding the way the have been composed, they are probably less interesting.

6.2 Conclusion.

Here we conclude the analysis of the pieces. I hope there were useful comments on the music proposed. I usually let music to speak by itself, but in this case I think it was necessary to spend some words regarding the process I went through to obtain this very particular kind of music.

Through the composition and the recording of these pieces I had the opportunity to experiment most of the different approaches I came up with, after various consideration about the material offered from the system.

As already pointed out the aesthetic of this music is not at all relied to the method in itself *which is only a tool* to be able to create music with new features. This music reflects, my personal way of approach music in general. Therefore I can find similarities with other works I have done without quarter tones.

This method opens up to a lot of different possibilities which I hope will be taken from different composers and improvisers in different genres of music. It allowed me to explore new sonorities which I was not able to imagine before developing it. Thanks to it I had the opportunity to blend together Arabic music with the tonal system and the jazz language regarding improvisation as well as the experimental European music concerning the form.

I think that this work demonstrates that quarter tones can be considered an extension of music possibilities independently from the traditional music which already use them.

I repeat here that I evaluate this method as successful, since the compositions proposed (even if they might not be the best result)sound coherent and pretty original I prudently affirm. Furthermore the method has indeed brought some musical result and it has been useful into the creation of music which has not only experimental intention, rather also *artistic ambitions.*

Came until here we have now concluded our treatise regarding the method and its concretization in music.

We therefore move forward to some practical consideration and further questions regarding the whole research. Last reminder: other music material can be found in the addendum (scales and graphics).

Chapter 7

Instrument practice considerations

We are now approaching the last sections of the work. After the conclusive chapter in which I will discuss the research itself, I Here the report of some issues I have founded into the practical act of quarter tones playing guitar: I describe the system I used to play quarter tones on it and I report some acoustic issues I found in playing quarter tones.

This chapter is thus referred to the craftsmanship side of this work: here I share the process which has led me to the actual possibility to play with this twenty-four notes system.

As the reader will notice, this chapter is more a report of the done activities, therefore the style in this chapter will be more like a diary with some explanations of practical operations.

7.1 Playing quarter tones on the guitar

The first step towards the creation of a music which makes use of intervals different from the half step is logically to be able to play them and thus to listen to them. Since the context in which this research has born and developed is practical, to possibility to hear quarter tone sounds and the possibility to play with them is clearly vital.

Therefore the first issue I faced was precisely the instrumental one.

Some instruments are built according to a system which allows to intonate all kinds of intervals; some others are not. Fretless string instruments (as violin or Arabic Oud) do not have fixed intervals on the fret, therefore on them it is possible to subdivide the string in all the possible portions and then to intonate whatever kind of interval.

Strings instruments are not the only one, for instance also the voice also allows the intonation of all the intervals. Furthermore, wind instruments have always or nearly always keys which determine a defined number of notes, but they are very sensitive to the lips of the player who can then easily vary the intonation.

Finally, the string instruments with frets and a fixed-intonation mechanics (as piano and harp) are actually the ones which do not have any possibilities to intonate intervals which

are different from the set ones.

This seems a big obstacle thus for a guitar player who wants to approach quarter tones. I clearly had to face this issue and elaborate a peculiar system to be able to play them on my fretted instrument.

Weirdly this obstacle became a very encouraging aspect of my instrument which made me stronger in the belief that this research could have reached some consistent artistic results.

On fretted string instrument such as the guitar it is possible to modify the intonation thanks to the technique of the bending (to pull the string up, down or left) which increase the pitch directly proportional to the pulling of the string. Nevertheless, the first problem I found working on intonate quarter tones on guitar was to be precise, exact and accurate. If it possible to intonate quarter tones with the bending technique, it anyway does not guarantee precision in intonation. Moreover, the bending praxis also forces to use a particular technique in order to obtain quarter tones. This of course limits the usage of quarter tones under particular conditions on the instrument.

For instance this approach would not have allowed me to play harmonies with quarter tones, no chords would have been possible.

It is now clear to the reader that I had to find a different way to approach the matter, so I focused on the tuning process. When you can not change the frets, you might change the pitch directly from its root: the string.

Thanks to a different tuning system I managed to obtain the perfect intonation of quarter tones without giving up the twelve half steps and being free from the bending technique. As said before, the biggest obstacle became the biggest fortune: a guarantee of perfect intonation on quarter tones as well as on half steps derived intervals.

I took in consideration different methods for alternative tuning, everyone with a different result, but they always carried an issue: I wanted to have the possibility to play all the notes existing on the register of the guitar (or nearly everyone) both *in half tone tuned* and *quarter tone tuned*.

For the ones who does not know it, the guitar is fretted such that neighboring frets produce a half-note change in pitch. Although, there are repetitions of the same note in different positions of the fret.

I now propose an example. Traditionally the guitar is tuned like this (starting from the sixth string, the lowest one): E1, A1, D2, G2, B2, E3. [1]

Considering what has just been told about guitar fret, let us know consider for example the second string: B2.

[1] where 1 means the starting octave, 2 to higher one and so on.

If we go one fret further from there we will obtain: B2 + 1St = C2
Going on like this we will go on with the chromatic scale: C2, C♯2, D2, D♯2,E3. The E3 which belong to the fifth fret of the second string is exactly the same of the one belonging the first string without frets and so it is for the ninth fret of the third string (G2 + 9 St = E3).

(See ⋆ *Video 1*)

This feature, usually used to verify the correct tuning of the instrument, has been the starting point towards the creation of a new tuning system with quarter tones. According to the law of equality between *free string* and *fretted string* I started to de-tune the first string (the highest one) a quarter tone lower. Consequently, on the fifth fret of the second string we will not find anymore the equality E3 = E3, but there will be E3 (second string) ≠ E3 *lowered* (first string).

(See ⋆ *Video 2*)

Moving on one fret to the next one it will this way be possible to find all the couples of keys intonate/quarter tone lower.

For example: sixth fret second string = F3; first fret first string = quarter lower F; and so on.

Then, of course, thanks to this system it is possible to obtain a double chromatic scale perfectly pitched (frets will maintain their capacity of moving by half steps) starting from the values 5-0 (fret wise). Explaining it better: starting from fret zero, string one, we will obtain the chromatic scale from E3 to B3 (stopping at fret seven) all *quarter tone lower*; on the other hand starting from fret five string two we will obtain again the chromatic scale from E3 to B3 (stopping at the fret 7 + 5 = 12) but this time intonate. There will always be a couple of notes intonate/quarter tone lower every time we go up one string and five frets (E = 5/0; F = 6/1; F♯ = 7/2 . . .).

This allows to have a perfect pitch on all the keys of the scale of 24 notes which I have called *Ultra-chromatic scale*.

(See ⋆ *Video 3*)

Even the greatest inventions always carry problems. Then, clearly the fingering will of course be not really comfortable within this tuning (it is always necessary to skip five frets in order to obtain the next tone). However this tuning system is surely the only one which allows to have the whole twenty-four intonate notes on the guitar; therefore I stuck to it.

Going on with the tuning of the remaining strings we face and solve the problem of playing chords.

Going up with the strings we always find the same kind of relationship S1 = S2 + 5f [2], the only exception is between S2 and S3 where the relationship is +4f. S2 = S3 + 4f.

Thereby, it is necessary to tune the third string referring to the first one in order to keep the equality of tone between first and third string. This way we will then obtain the following relation: S1 = S3 + 9f.

The E3 quarter tone lower obtained plucking the first string without fretting will be exactly the same of the one on the third string ninth fret. The fourth string will be tuned then according the second one. This allows to keep the same kind of relationship of before: S2 = S4 + 10f [3]. And so we tune also the fifth and the sixth string.

This way makes it possible to obtain the whole *ultra-chromatic* scale in more than one octave, and moreover we also got three strings perfectly pitched with a quarter tone distance in between.

This peculiarity enable to create three voices harmonies and chords perfectly pitched according to the rules of the conventional twelve keys music, also starting from a quarter tone note. It means regular harmony in not regular places, together with a new harmony with quarter tones.

As observed previously in chapter 6, this peculiar characteristic of the tuning system coincides with some compositional items I have developed into the composition. Of course it is not relevant to determine if *it was born before the egg or the chicken*, however there is of course a deep link between the practical possibilities of the instrument and the theoretical items I have developed.

Again, since this research is meant to propose some new kind of compositions and performances, it is strictly necessary that everything it is postulated into the theory of the methodology can be later on performed.

Since I had my guitar to investigate the sounds and generally this subject, definitely part of the choices undertaken are related to the possibilities of the instrument.

Thus I do not exclude that the same method could have been developed differently from another musician or even from a *different myself* if I would have approach the instrument issue in a different way; proposing different solutions.

Going back to write upon the tuning system chosen, this tuning system opens up to a lot of different possibilities in playing quarter tones, without excluding the opportunity to play conventional music.

This is also a very important aspect of this tuning, it also allows to play half-step con-

[2] where S means string and $5f$ means 5 frets

[3] there is one fret more here because of the normal tuning of the guitar

structed music. Thus it does not imply a complete negation of all the music *bagage* I have developed among my music studies and music experience. This tuning system has therefore allowed me to escape the trap of modernism and technique research that a quarter tone research would have implied.

Thanks to it the music has still the possibility to easily carry expressive contents, along with a strong research drive.

Curiously, it is also similar to the double keys pianos used by Charles Ives and Alois Hába in the early twenty's century.

Nevertheless, as it is clear, this new way of playing the guitar obviously imply new difficulties both technically and orientation wise (it is needed to learn again the whole keys positions on the fret) but at the same time it leads to completely new possibilities together with great versatility.

As said before, this tuning was the starting point of the research. This choice has partially led its development and it gave me the possibility to test the whole methodology . [4]

Passed this last section of the research, I now go further with the last part of it: the evaluation *postequam.*

Might the reader then follow me into the last step, the act of looking backwards and being severe (or maybe not too much?) with the work done.

[4]Regarding the other instruments, I had an experience with colleagues whom play saxophone, double bass and harp. This last one seems to host well the method proposed in this thesis.
Concerning the other instruments I just want to underline that regarding the saxophone I worked on fingerings with my colleague Andrea Ayace Ayassot, in a direction similar to the ones showed in this online video: Philipp Gerschlauer. *128 notes per octave on Alto Saxophone.* Available at: `https://www.youtube.com/watch?v=lGa66qHzKME` [Accessed 12 May 2016].

Chapter 8

Conclusion and further questions

Finally arrived to the end, I here evaluate the research. I compare the searched goals with the actually achieved ones and I try to analyze objectively the process I went trough, trying to understand what went good and what less, in order to imply further reflections upon the subject.

Furthermore I also propose possible further developments of the research, gathering few point already emerged among the whole writing.

8.1 Discussion on the work, general considerations.

As the reader has noticed, this research is constituted by multiples layers of work.

The principal one consists into the proposal of a theoretical method which can allow the composition and performance of music whit quarter tones, the secondary one is the discussion of a syntactic approach towards music language analysis (and the reflections which come out from this issue).

Regarding the main goal, this research had the proposed goal to determine a *traité* able to expand harmony to quarter tone music. Regarding the accidental goal it had the ambition the take part into a big discussion which involves semiotics and linguistics.

Surely I can affirm that the main goal of this research has been accomplished. It has been composed and recorded coherent music making use of the propose the method which, from its part, has grant a lot of useful scales and approaches towards the treatise of quarter tone music independently from the analysis of existing tradition.

This specific point can obviously considered both as warranty of achieved goal (since this method has therefore proposed a surely new and independent way to approach the subject) and both a problem of the method itself.

This disjunction between method and literature might generate both some redundant result (some results already achieved by other traditions and systems will be found again

approaching the matter from this method) and both some purely theoretical results as well.

We might thus comment that the adoption of a method purely based only on geometric principles is not able to guarantee a decent musical result since, after all, music comes from culture and not from nature.

However I consider the artistic goals achieved with the recording a valid answer to this issue: even if I can definitely not affirm that every method can automatically generate artistic results, I observe that this one in this particular case has done it.

On the other hand, regarding the treatise of the linguistic approach to music, I have to observe some problems.

First of all, since this research has not as main goal the approach to these issue, the problem has not been investigated in depth as it deserves it.

Many assertions should have been argued more in order to reach a deeper level of rational achievement of them, by the way I also think that a lot of interesting points have emerged. Furthermore I truly believe that those issues might be developed in further studies bringing up very interesting issues.

In particular the analysis of music with the usage of syntactic procedures might be (made the premises that have been made in this research) very promising.

Moreover regarding the method propose, I want to say that, first of all, this research has been a really stimulating process, which has led me to discover new aspects of music and guitar. The method proposed is of course not the only possible one. This research is not meant to be considered a book for the good composer, it is rather a tool to create music, in the same way as tonality and Schoenberg's dodecaphony systems are. After all, this method belongs to the base of music syntax same as dodecaphony does. They both deal with the primary organization of notes, they define rules for the construction of scales and harmonies. Therefore these kind of methods are involved into the composition of several different pieces of music. When we compose a tune using the dodecaphonic system we stick to determinate rules; but at the same time we compose many different pieces that will be related but different. The same with this method: it provides scales, harmonies and rules but it does not guide us towards the whole composition process.

I hope that this method can be considered and used by other musicians whose can express their own personal voice within its use. This way of approaching microtonality differs from the many others we can find in classical European music. Very often researches of new systems to organize music matter (especially after the 30's) start from the analysis of sounds from a scientific/objective point of view. They start considering sound as a physic event and then they organize the components of the sound phenomena. Those system are then often based on the analysis of sound, its spectrum, its dynamics and so on. They do not organize those elements into musical notes, they work with the primitive matter of sound finding completely new ways of thinking upon music. They do not work within a context of

limited number of equal object (music notes) and their possible permutations. Digital music went very deep into this aspect of music research. It created a lot of new systems to organize sounds without music notes. There are very few researches on new systems which stick to musical notes and which investigate the possible permutations and orders of meaning of a number of discrete objects. After Schönberg and Henry Cowell (among others) the musical research got far from the creation of systems made up of music keys. Especially regarding micro tonality, a lot of composers has approached it leaving the music keys system and getting closer to different notations. All these systems are based on a different conception of music which does not imply notes and their organization in scales and harmonies.

It is now time to face some problems which have emerged into the research process.

First of all, the biggest issue was to overtake the purely experimental frame of a structural research upon quarter tones into a contemporary music context. This problem of course still remains and it is difficult to be overtaken since it implies also a social acceptation of these new sounds. As long as quarter tones will conserve their *experimental* attitude in European culture, it will be hard to enrich them with artistic values. Of course this is a challenge which can not be even approached by a research like this one; time will judge. This research, as all the other ones of this kind, has only the purpose to *trough the stone into the pound*, to propose some new *idioletto* (quoting again Umberto Eco) into music culture.

Thus I will not say more regarding this issue.

Concerning the methodology and its problems, I might say that I found some difficulties into driving back the obtained results to some musically efficient item. By the way, a lot of results drove back to musical relevant scales already into the barely geometrical step. Moreover the *generative diatonic scale* deduction process has given me a very useful tool to approach the issue. Of course a lot of results has still to be analyzed and the system (as every system) has produced some scrapes, set at the far edges of the method. Nevertheless I have clearly adopted a selection of the material from the method, discarding those useless results.

More consistent are the practical issues.

It has been very difficult to reach a good level into playing quarter tones on the guitar [1] and furthermore it has been even more difficult to find some other musicians to play with this kind of music.

Nevertheless the recording proofs that it is not impossible to play the proposed compositions, even in ensembles.

I do not exclude to find some other colleagues which will be enthusiastic the same way I was when I decided to undertake this work and which therefore will study techniques to

[1] The reader might be curious to observe a solo performance as *visible* result of it in the following link: https://www.youtube.com/watch?v=27uqDRHU9yo

approach this method on her/his instrument.

The limitation of this method to the harmonic developing of the music is not a problem I think. In this work they have not been faced phraseology issues as well as rhythmic ones. Nevertheless I also think that this limitation has even helped the system to be possible to be used in various contexts, to be flexible.

Regarding the possibility the use this method into traditional context (such as playing solo choruses on the top of some jazz standard) I feel faithful and suspicious at the same time. Even if I truly believe that this would be possible, I also do not really feel the need to juxtapose these music approaches, so distant one from the other one.

I recognize the fashion of this possibility, but at the same time I do not feel the artistic need to proof it. Thereby I have not investigated this possibility and I leave this challenge to other colleagues interested into the topic.

Going back to the secondary goal of the research, I feel that a lot of interesting steps forward has been here proposed into the field. I therefore feel exited and really faithful towards the further developing of this comparison.

I can recognize myself the fruits of this syntactical approach in music and also into the twenty-four notes derived one, here proposed. I truly believe that the consideration of music as language and scales as a modular additive operation, have given a great contribute to the successful composition with the method.

Without all those reflections I am not sure that the usage of the method would have been the same fruitful as it has been.

I then globally evaluate this research as positive in its results.

I now propose a brief chapter in which I collect some of the possible further paths to be undertaken after this work.

8.2 Lines for further researches.

Here I propose some further researches that emerge from this work and which could be developed in further studies.

I individuate four further researches.

- **Research 1:**
  The first one regards a further research of the method proposed, extending the dimensions from one to n. The method proposed puts in relation twenty-four elements on the same dimension and it uses properties of two-dimensions figures to generate relations. This can be evolved into a space which consider timbers and rhythms features together with the notes. Three dimensions figures could be used to identify peculiar relations

between notes and rhythms, rhythms and timbres and so on.

- **Research 2:**
  It would be interesting to go deeper into the study of vertical functions (See 3.4.1). In order to increase the tools to generate harmony coherency into geometric systems.

- **Research 3:**
  The third line of further research regards the study of music that uses quarter tones and try to explain their scales with geometric properties. As we found the major pentatonic using a triangle property, so it might be possible to find correspondents geometries to other important scales. I propose here the study of the Arabic Maqām and generally music culture in Arabic culture in order to face different system to organize quarter tones and trying to imagine a general context that can link all of them. It could be interesting to go deeper into the study of music traditions which use quarter tones and of music syntax in order to combine them for a deeper understanding of music.

- **Research 4:**
  The third research proposed regards the study of music syntax trough the tools utilized by the semiotics and the linguistic. This is probably the most ambition further developing but I think it might be one of the fruitful. It is interesting to go deeper into the functioning of formally organized systems, so to understand more and more of tonal and diatonic systems. Again, to go deep into the possible similarities between music and verbal language, starting from the syntax analysis (according to Philip Ball, [2] they belong to the same brain area).

[2] Philip Ball. *L'istinto musicale.* Bari: Edizioni Dedalo, 2011.

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Part I

Appendix

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Appendix A

Geometrical computation of tonal functions

Here there are exposed all the figures that has been developed to compute the set of absolute functions for all the partial symmetric schemas that we have proposed to analyze.

Four side figures

Here there are exposed all the four sides figures obtained by the absolute similarity principle closure.

Schema 1:

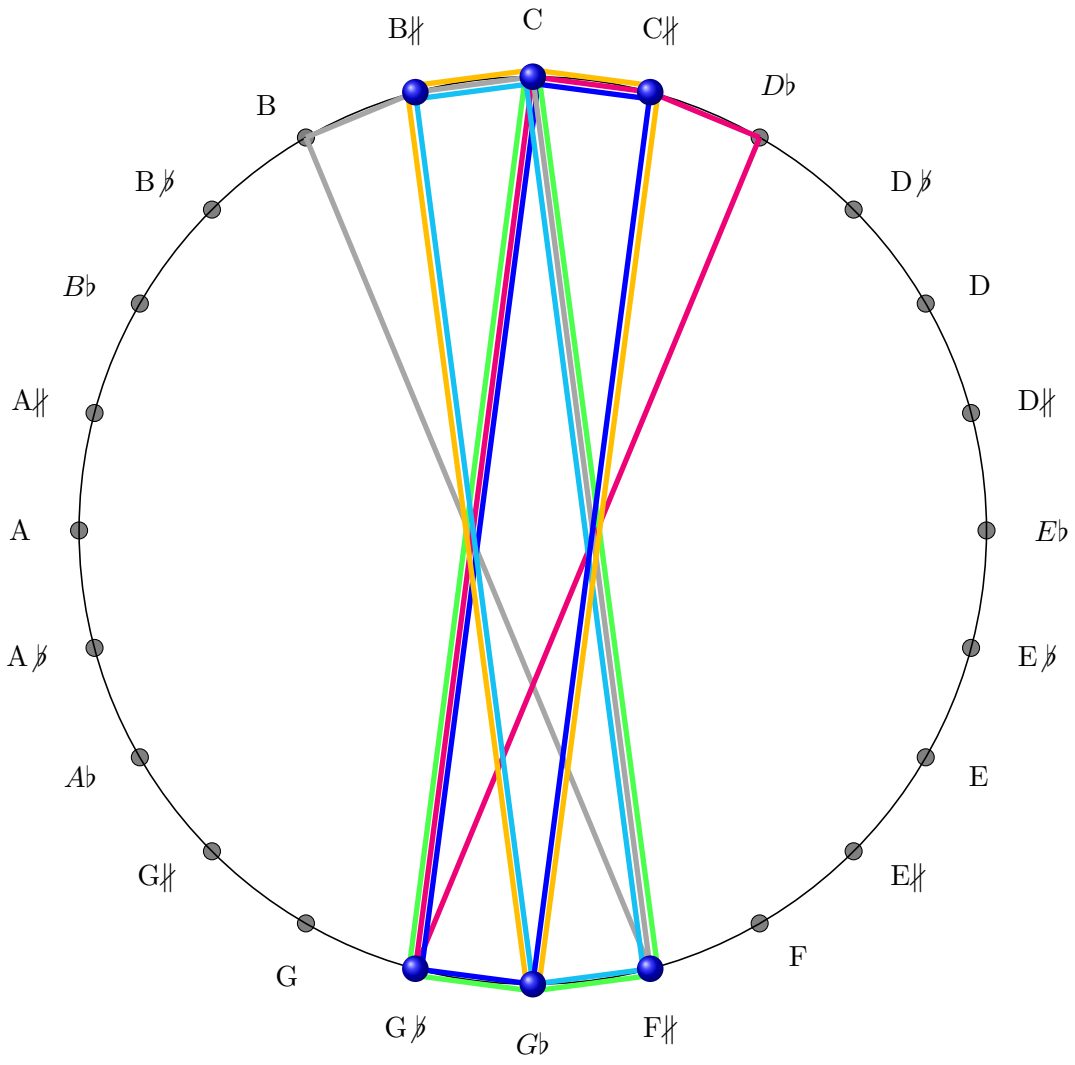

Figure A.1: Rectangles [1+11]

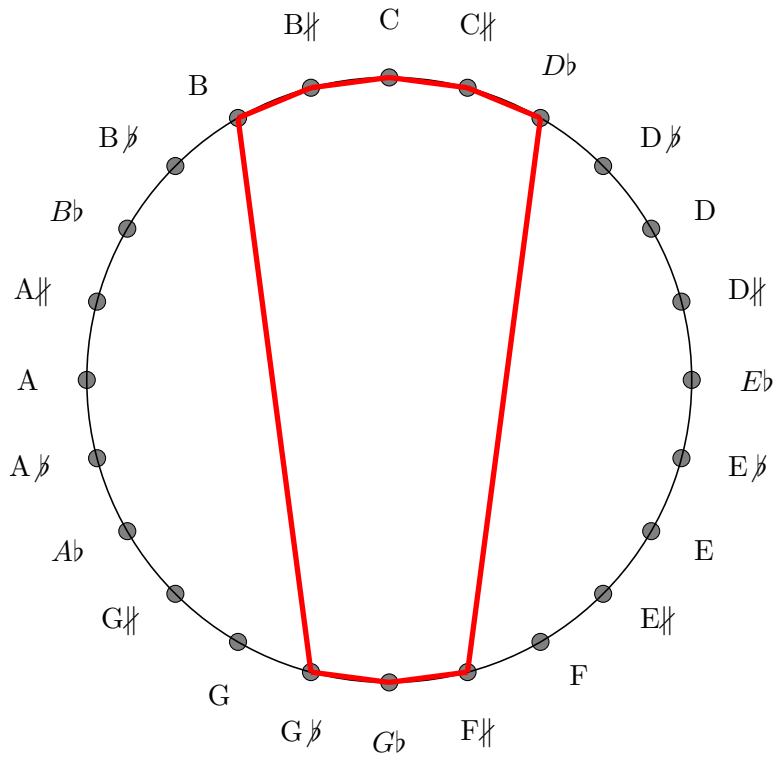

Figure A.2: The Absolute function for [1+11] rectangles

Schema 2:

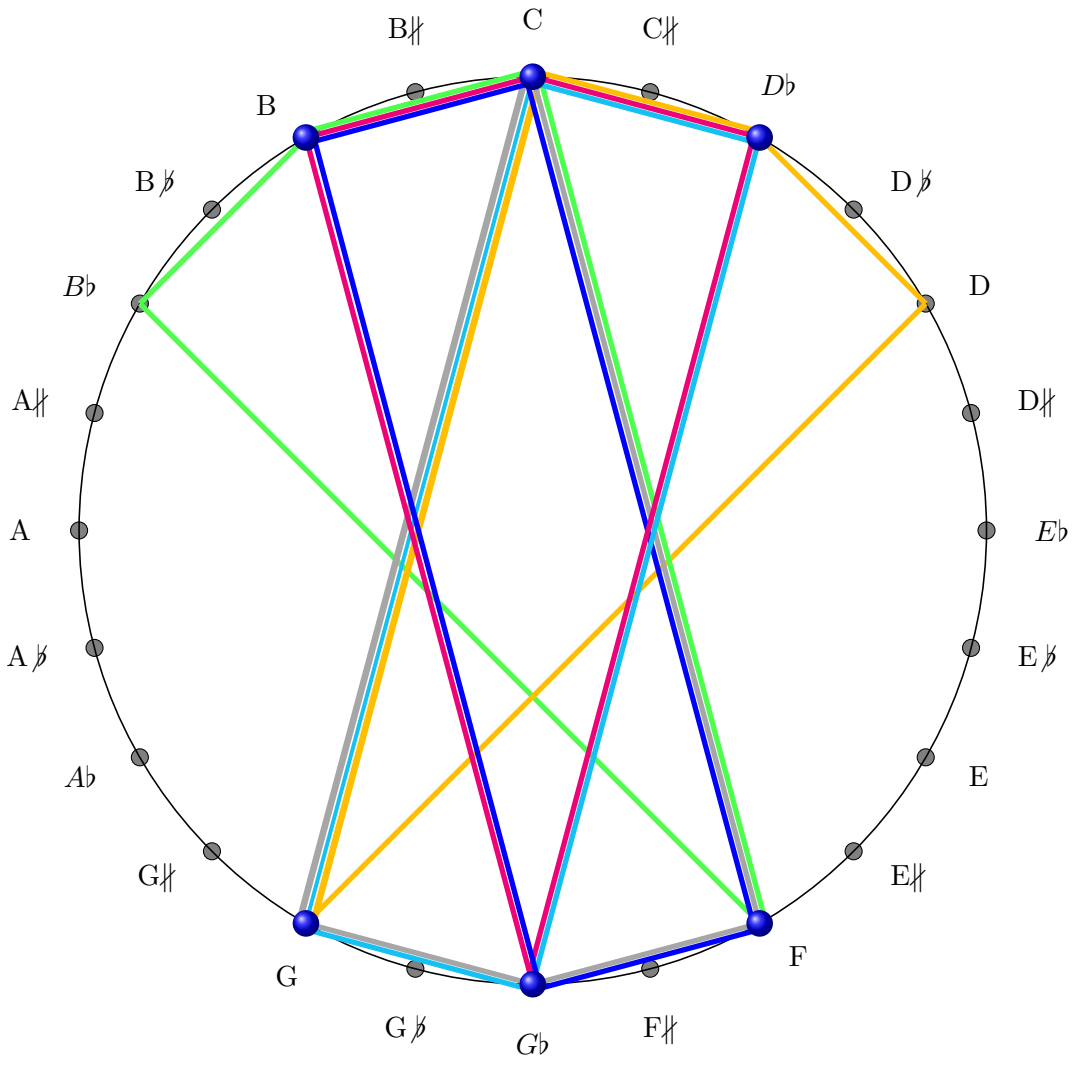

Figure A.3: Rectanges [2+10]

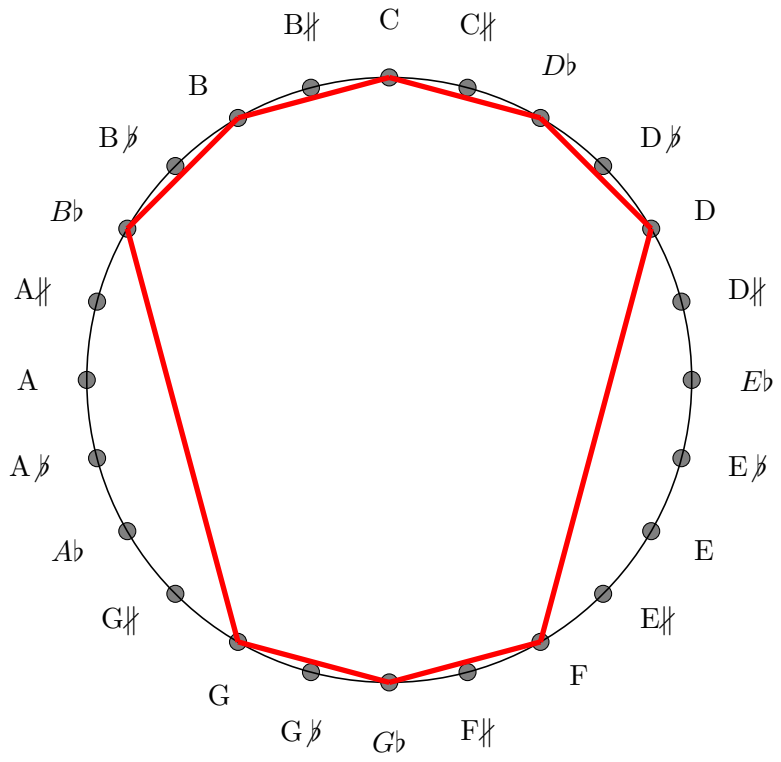

Figure A.4: The Absolute function for [2+10] rectangles

Schema 3:

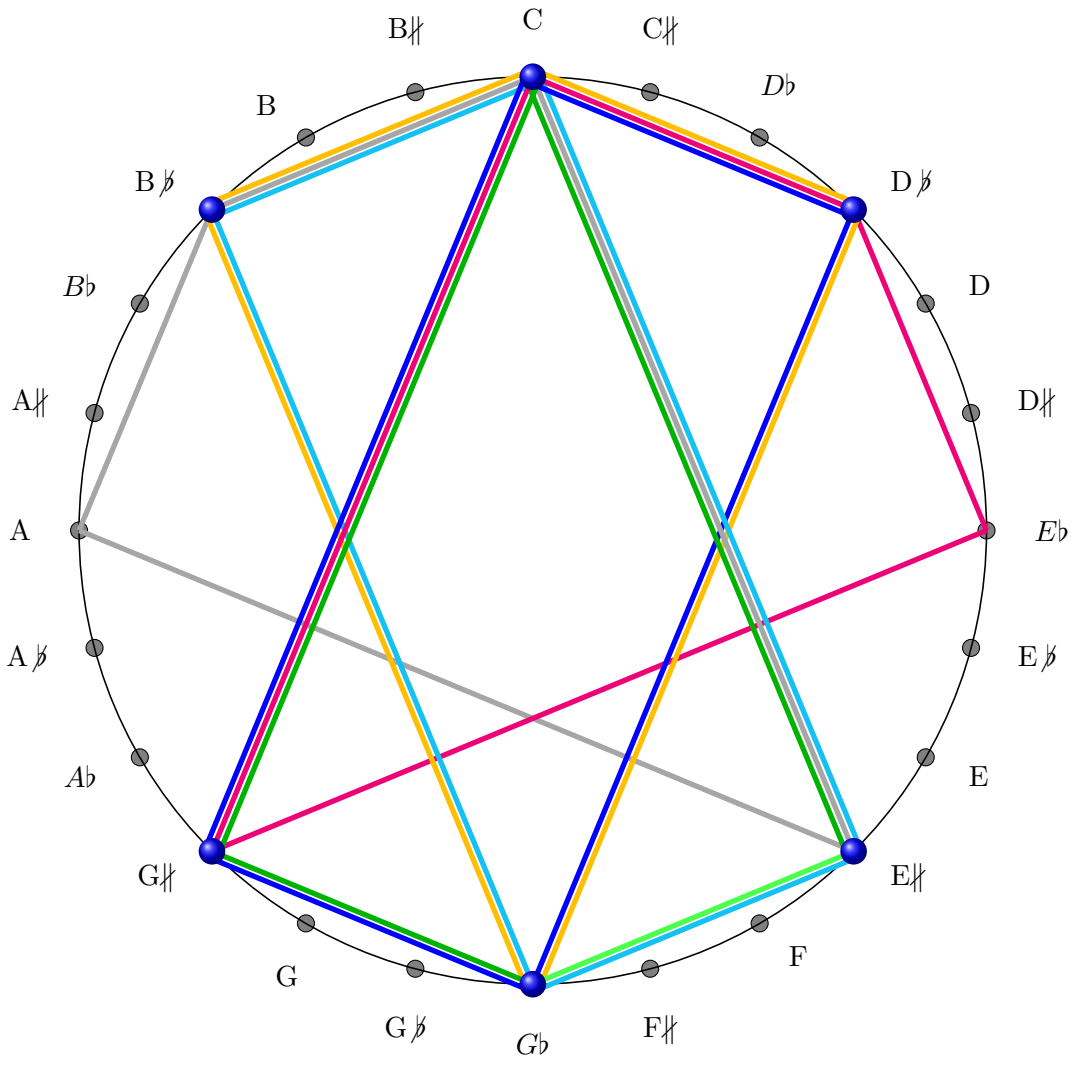

Figure A.5: Rectanges [3+9]

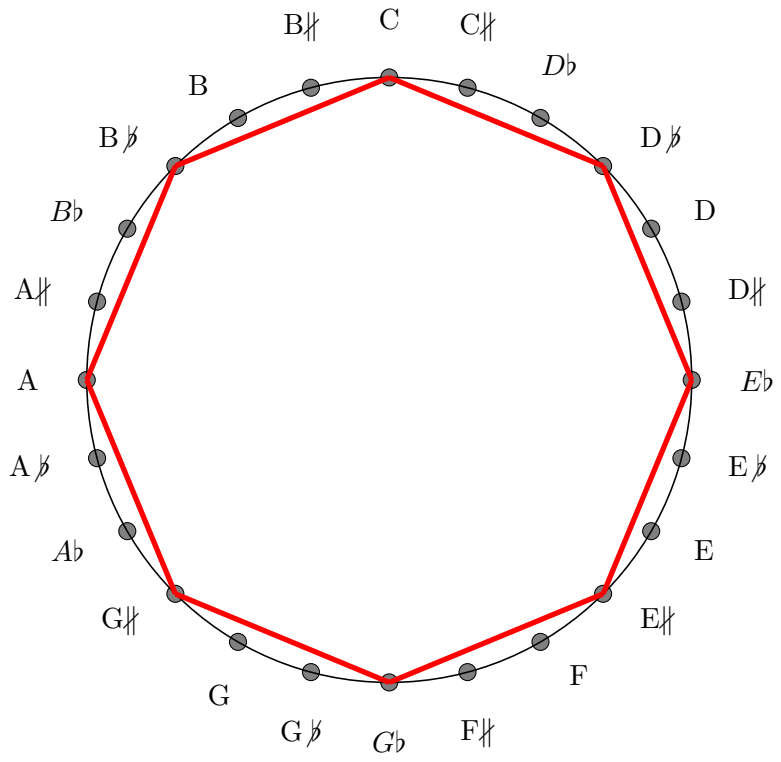

Figure A.6: The Absolute function for [3+9] rectangles

Schema 4:

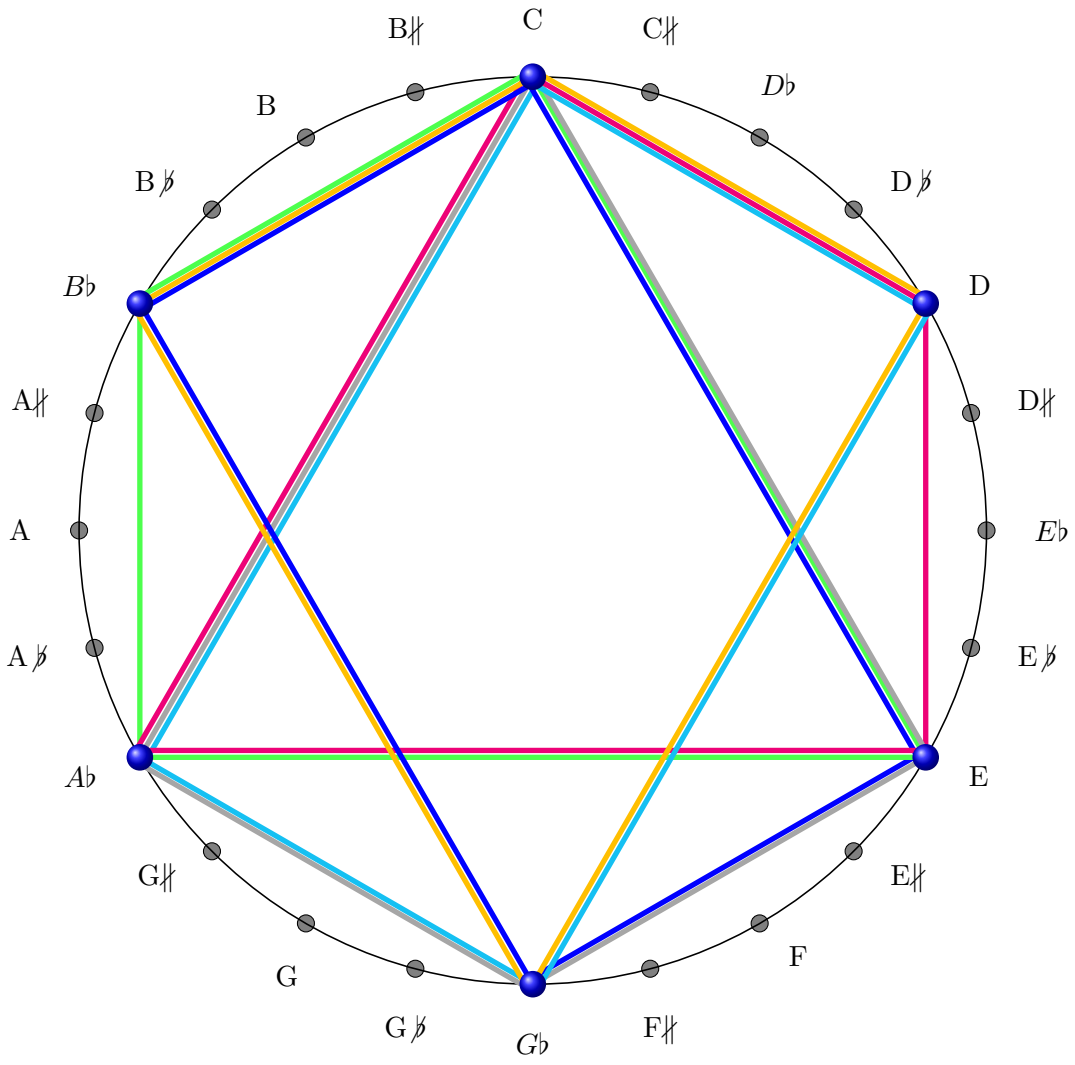

Figure A.7: Rectanges [4+8]

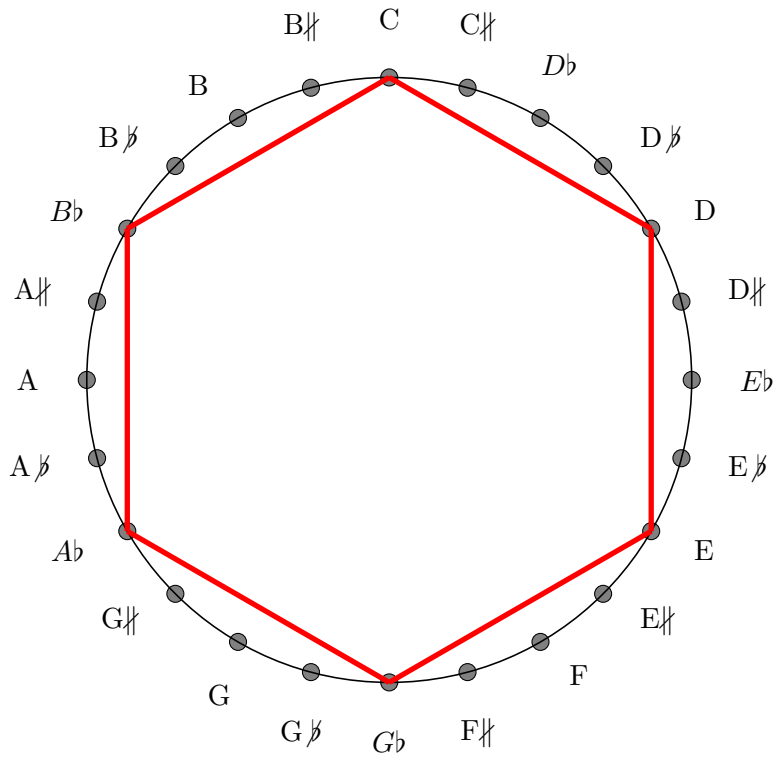

Figure A.8: The Absolute function for [4+8] rectangles

Schema 5:

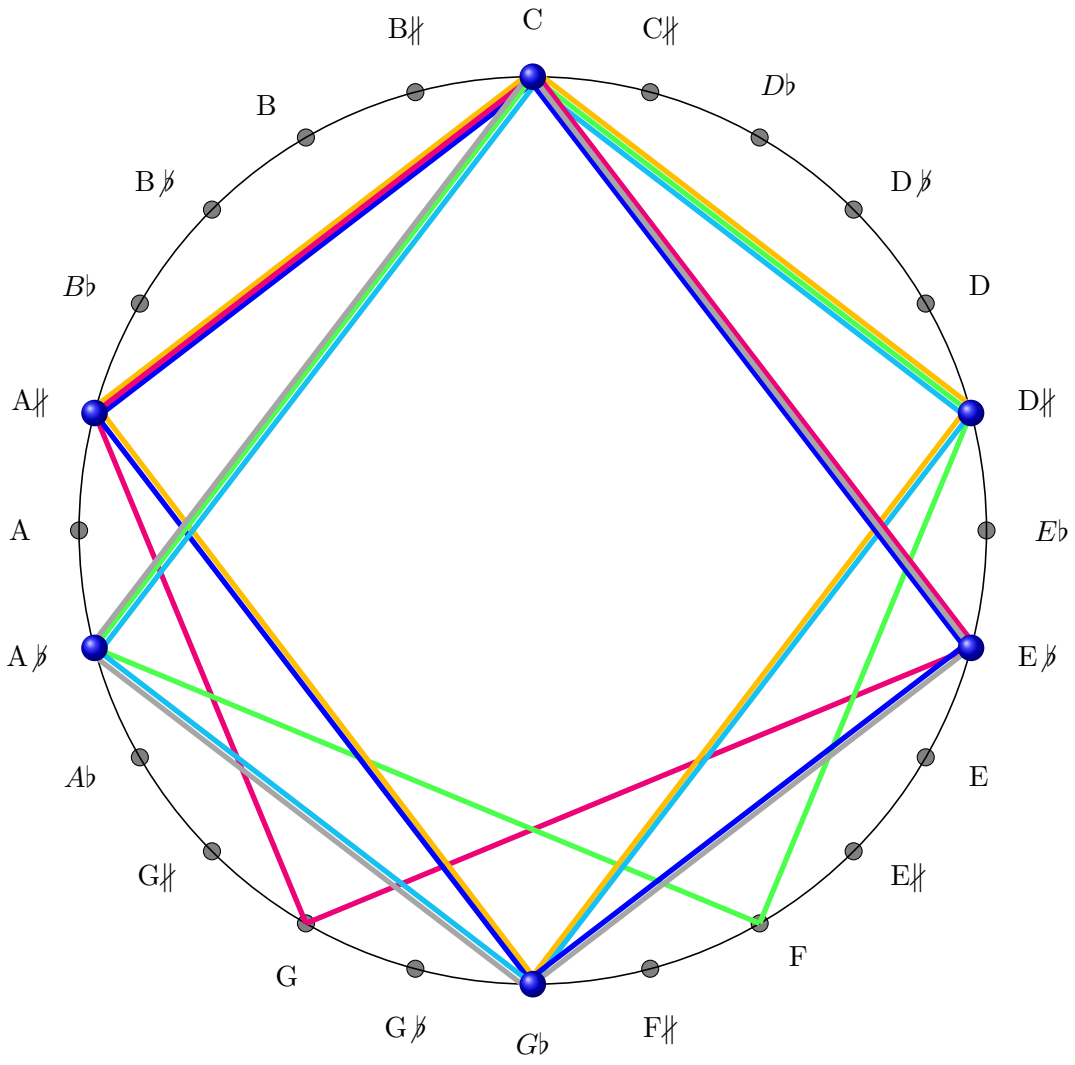

Figure A.9: Rectanges [5+7]

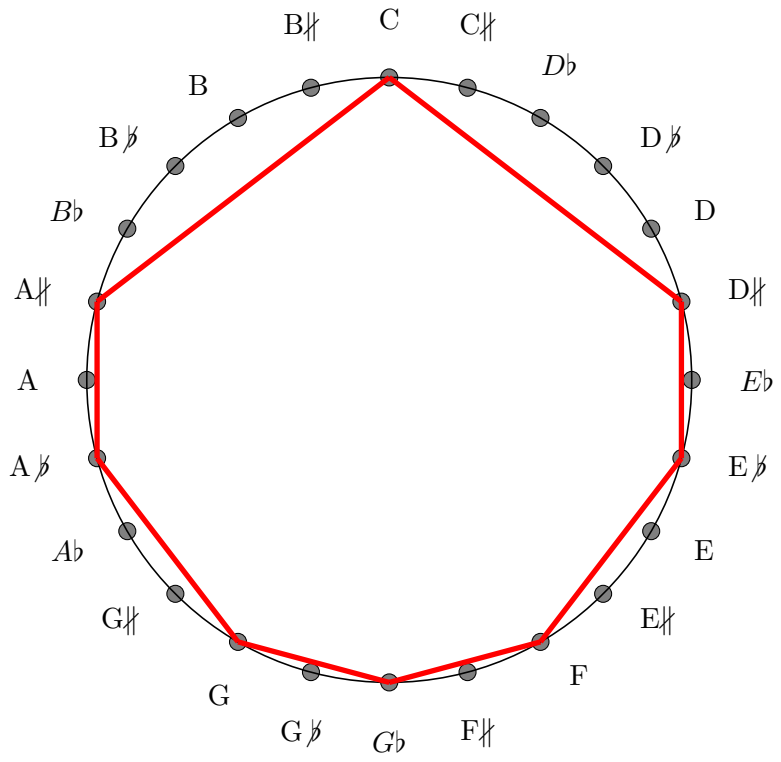

Figure A.10: The Absolute function for [5+7] rectangles

Three side figures

Here there are exposed all the three sides figures obtained by the absolute similarity principle closure.

Schema 1:

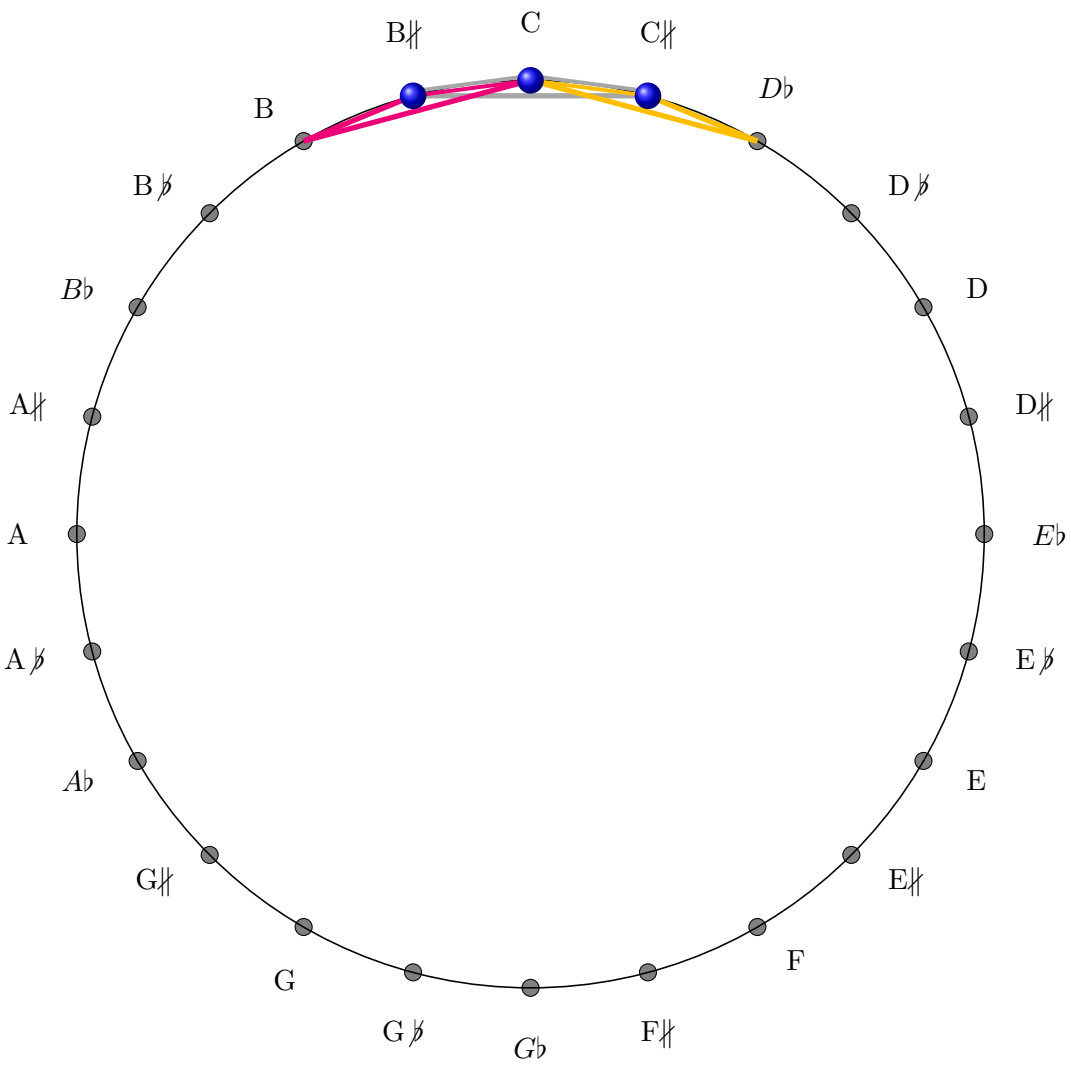

Figure A.11: Triangles $[\frac{2}{2}\ +22]$

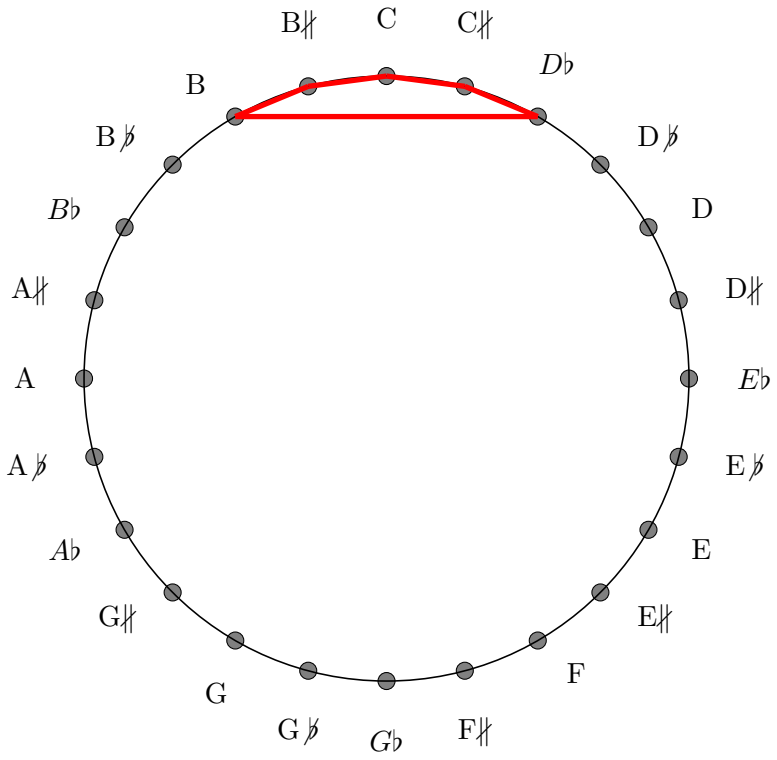

Figure A.12: The Absolute function for $[\frac{2}{2}\ +22]$ triangles

Schema 2:

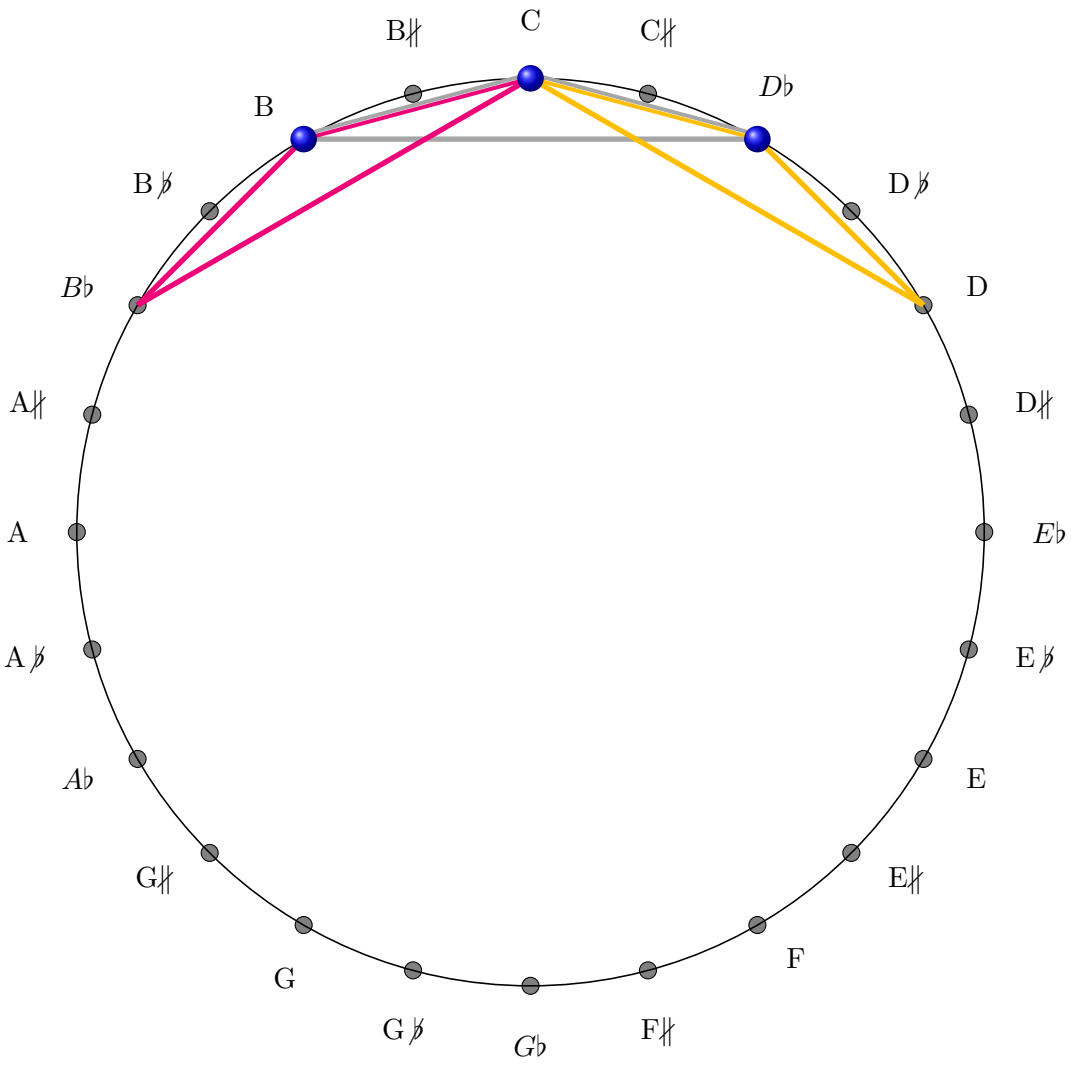

Figure A.13: Triangles $[\frac{4}{2}\ +20]$

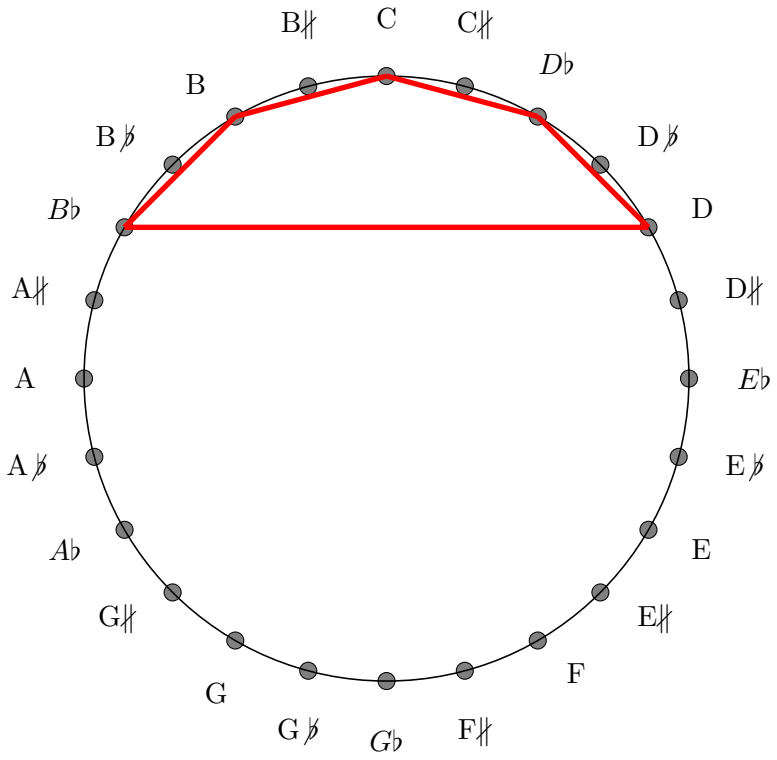

Figure A.14: The Absolute function for $[\frac{4}{2}\ +20]$ triangles

Schema 3:

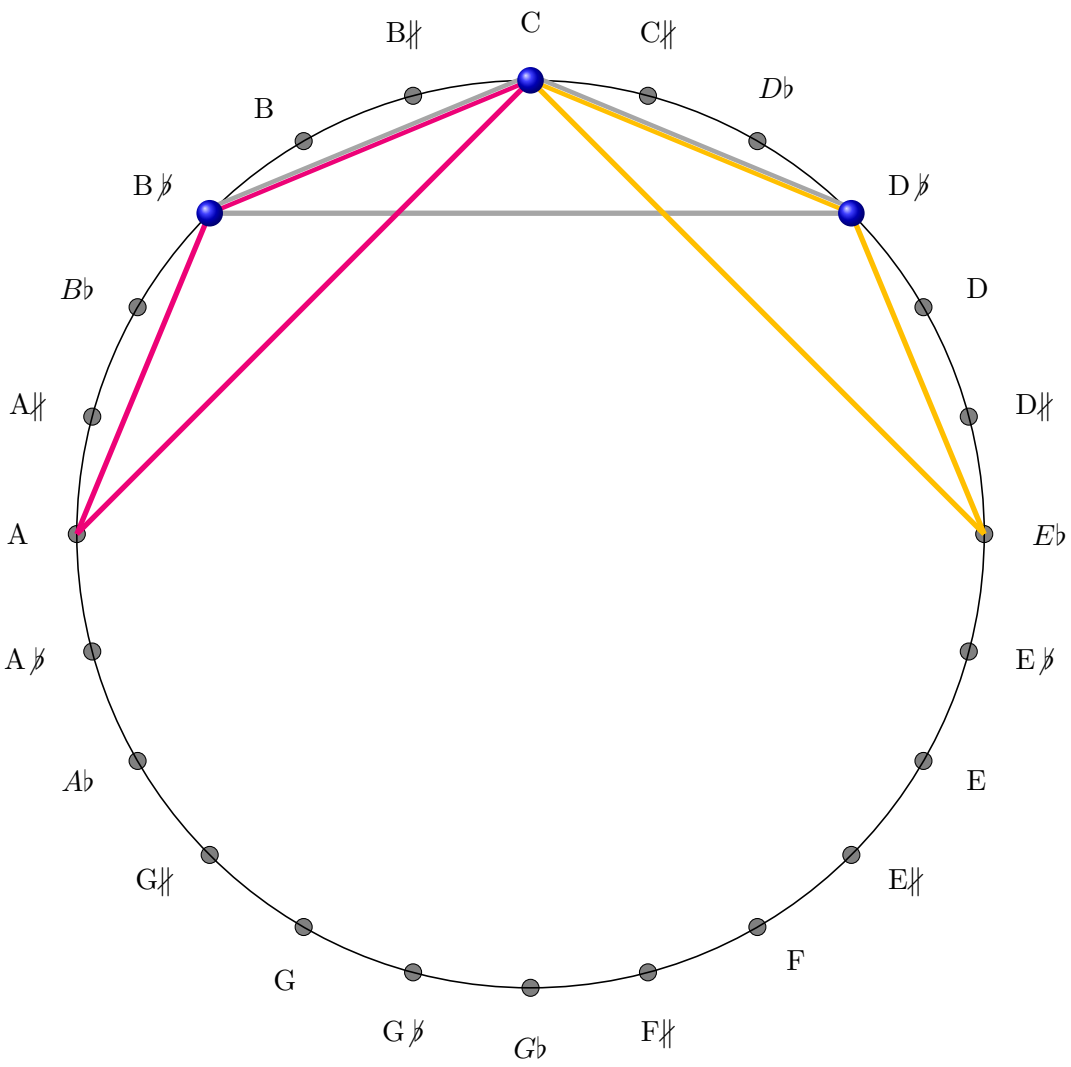

Figure A.15: Triangles $[\frac{6}{2}\ +18]$

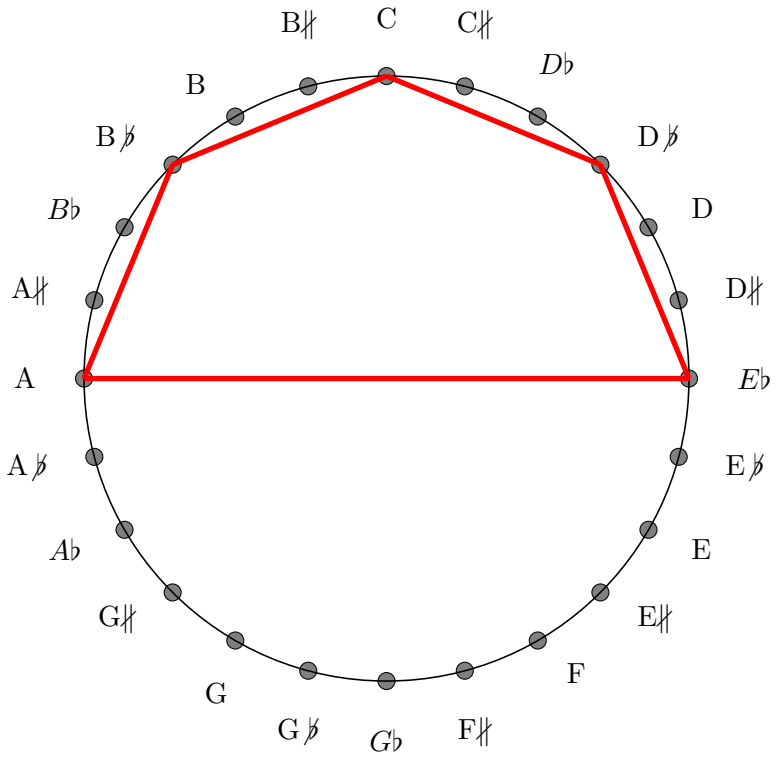

Figure A.16: The Absolute function for $[\frac{6}{2}\ +18]$ triangles

Schema 4:

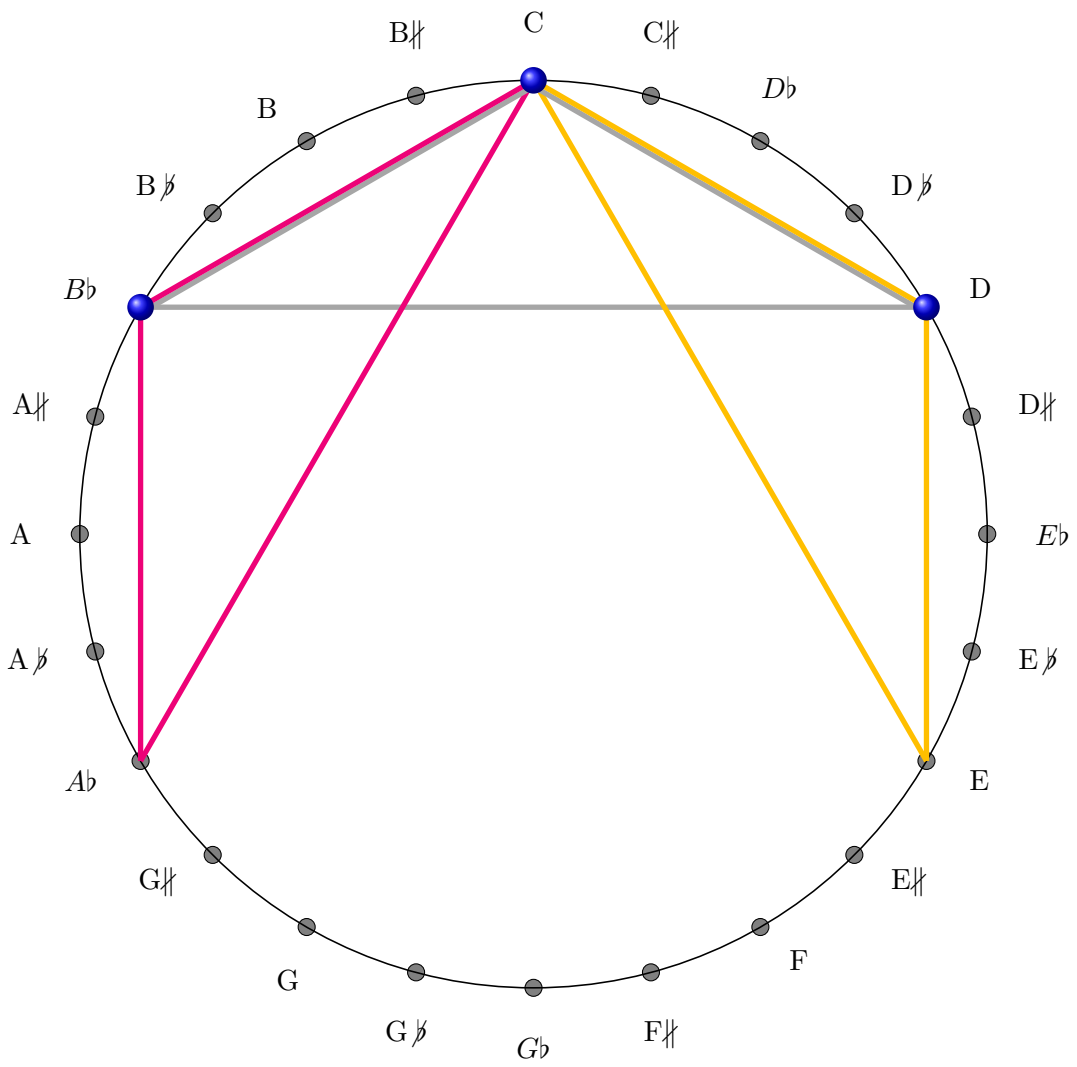

Figure A.17: Triangles $[\frac{8}{2}\ +16]$

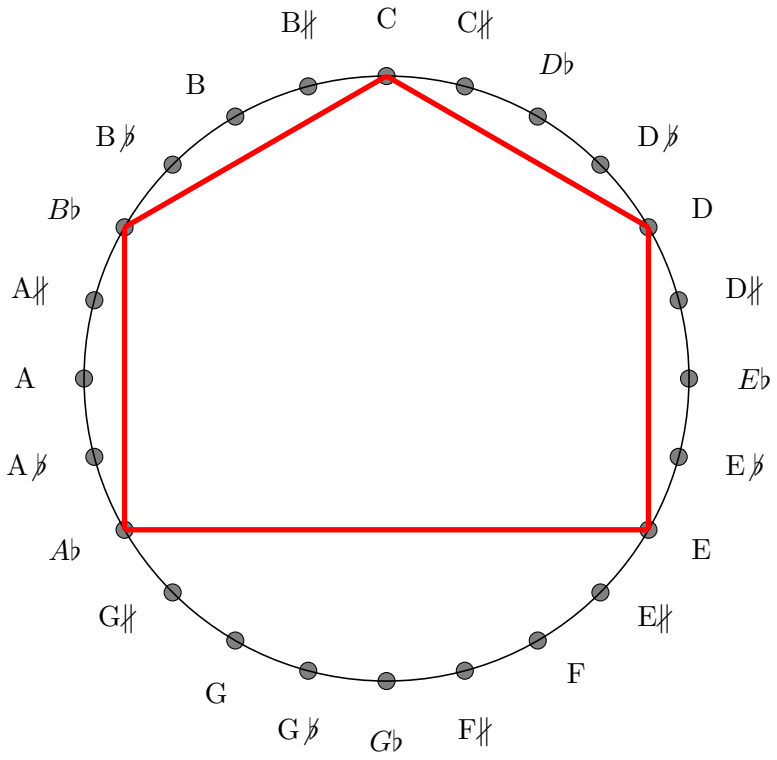

Figure A.18: The Absolute function for $[\frac{8}{2}\ +16]$ triangles

Schema 5:

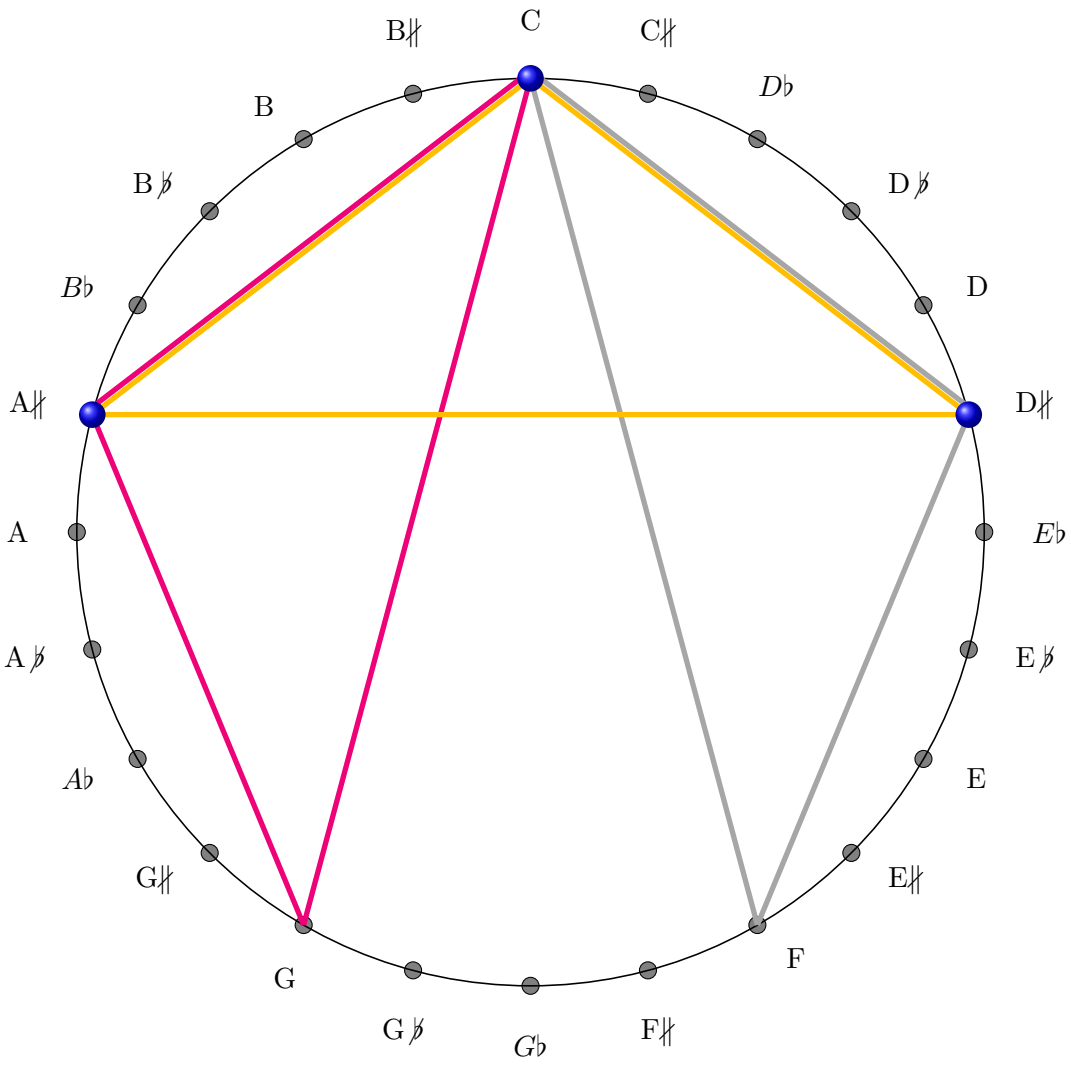

Figure A.19: Triangles $[\frac{10}{2}\ +14]$

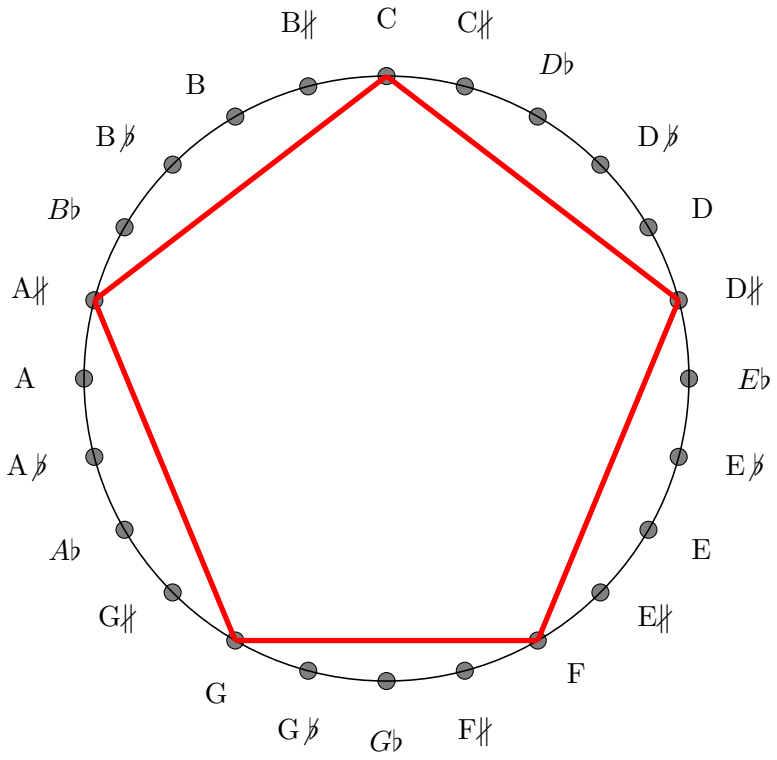

Figure A.20: The Absolute function for $[\frac{10}{2}\ +14]$ triangles

Schema 6:

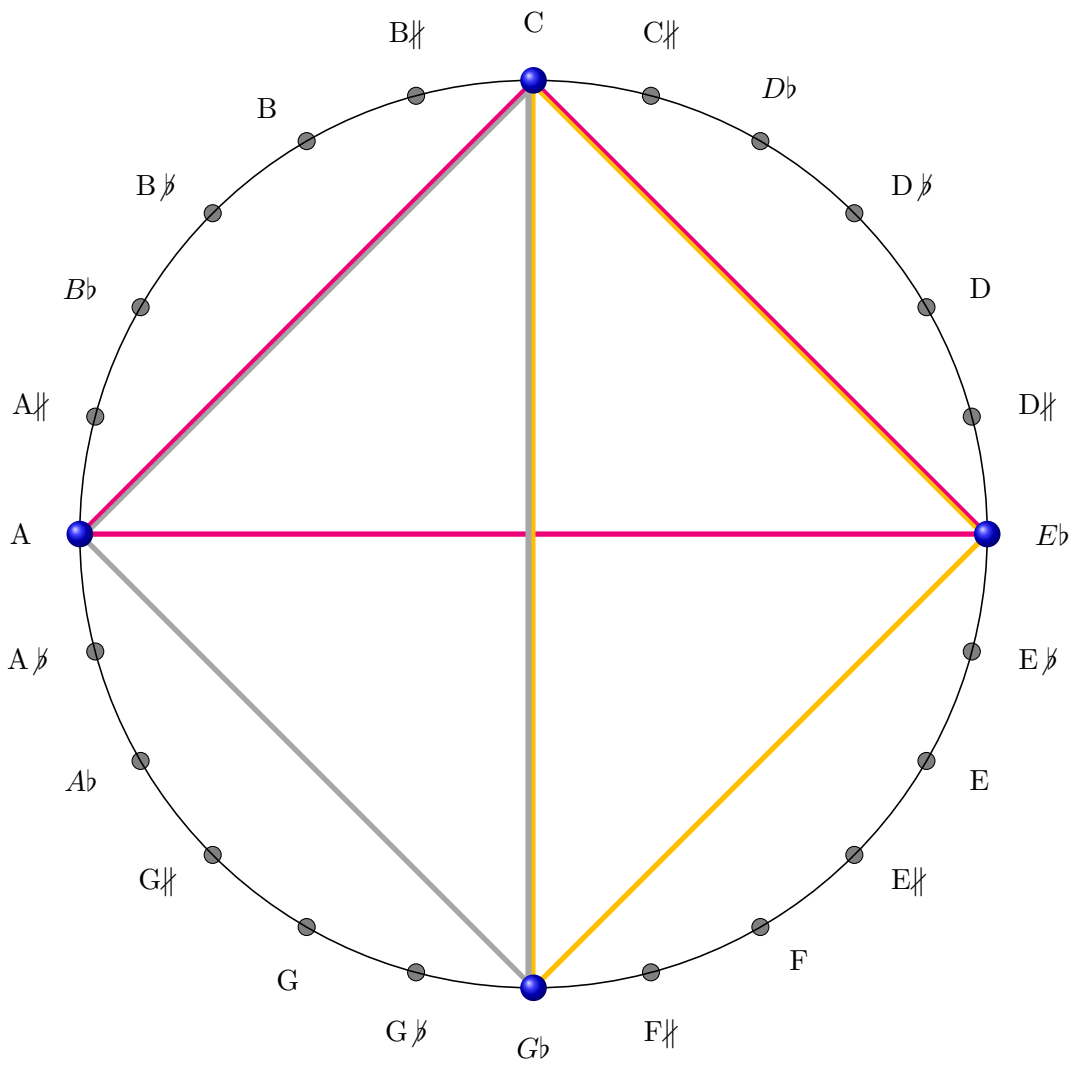

Figure A.21: Triangles $[\frac{12}{2}\ +12]$

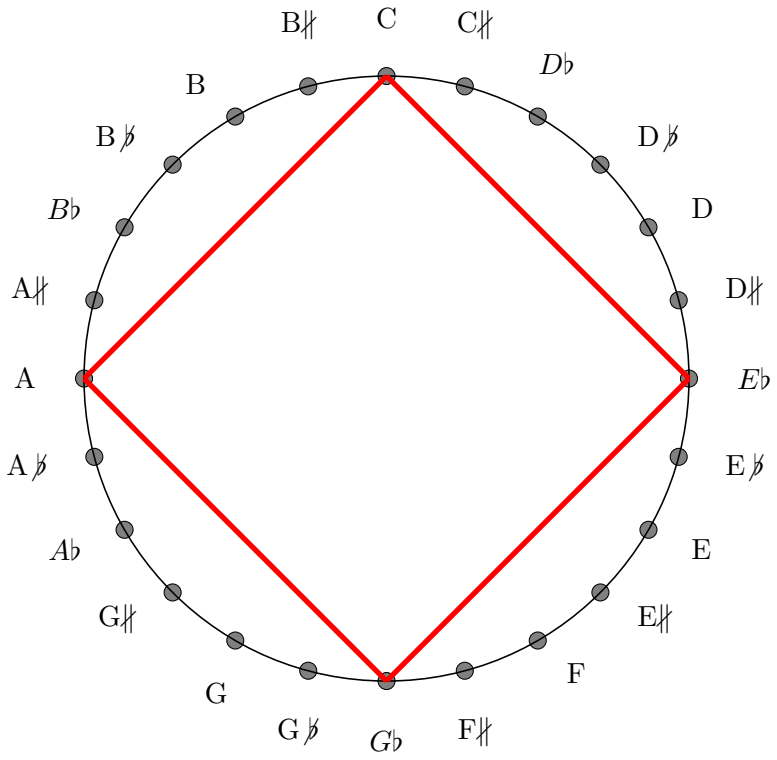

Figure A.22: The Absolute function for $[\frac{12}{2}\ +12]$ triangles

Schema 7:

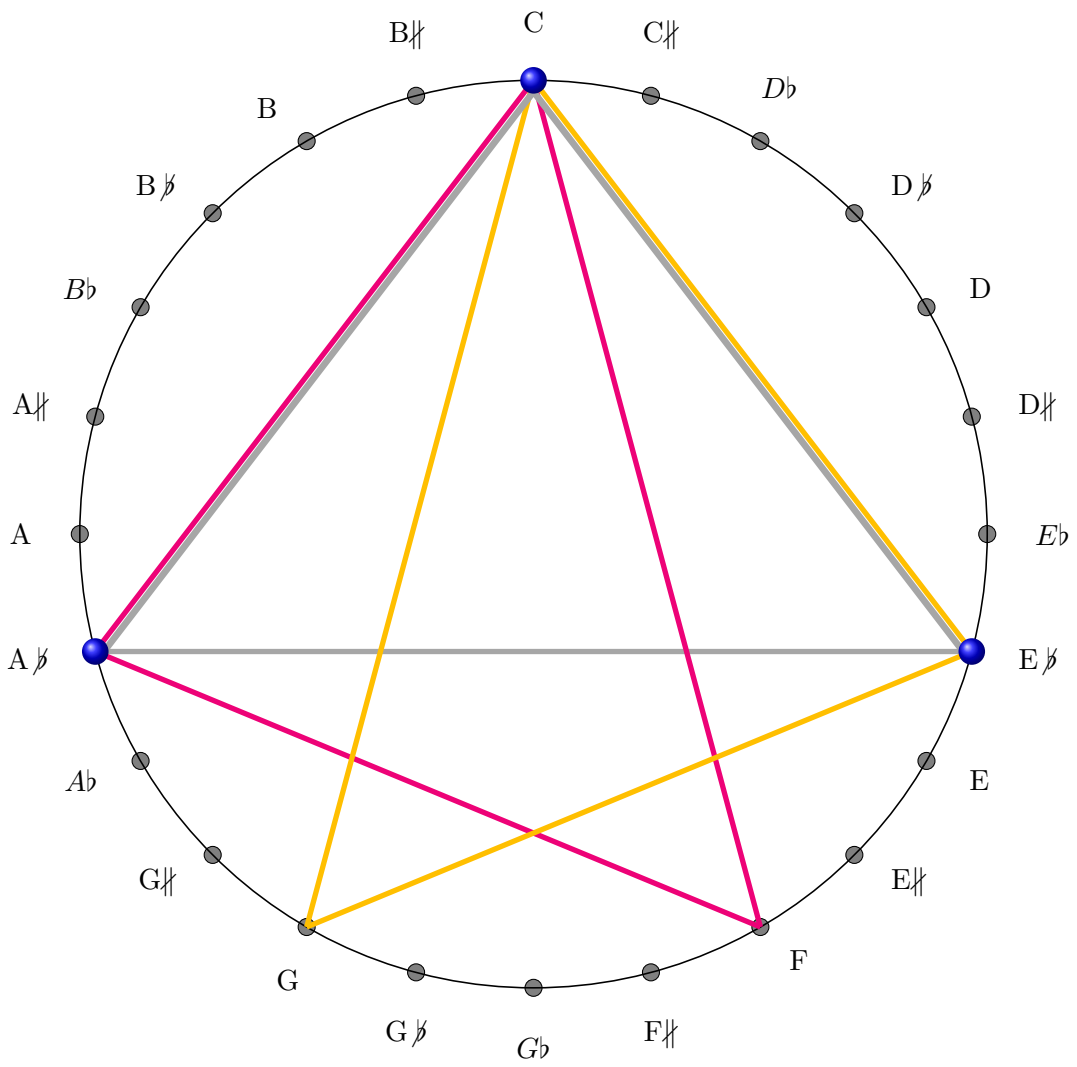

Figure A.23: Triangles $[\frac{14}{2}\ +10]$

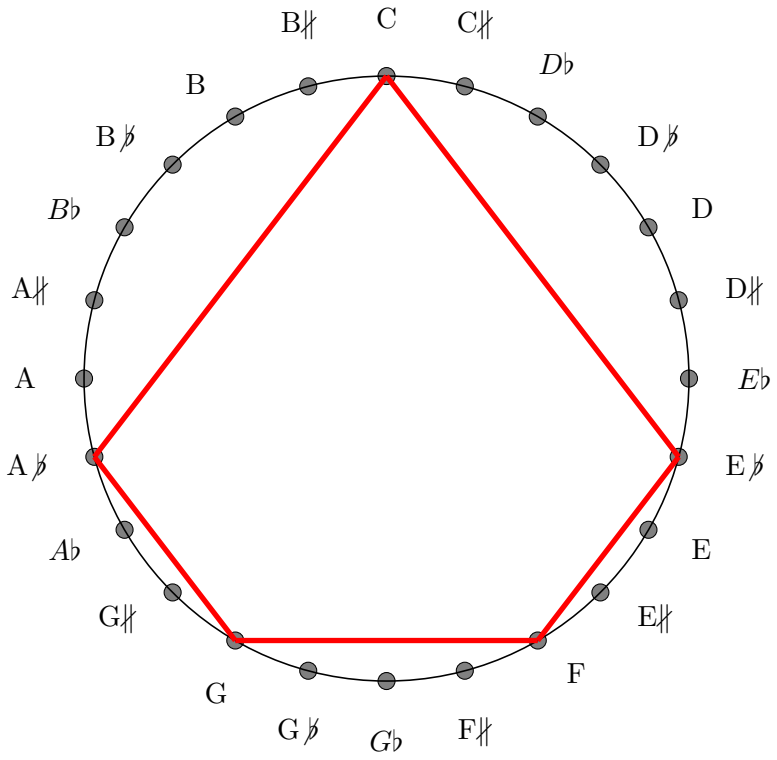

Figure A.24: The Absolute function for $[\frac{14}{2}\ +10]$ triangles

Schema 8:

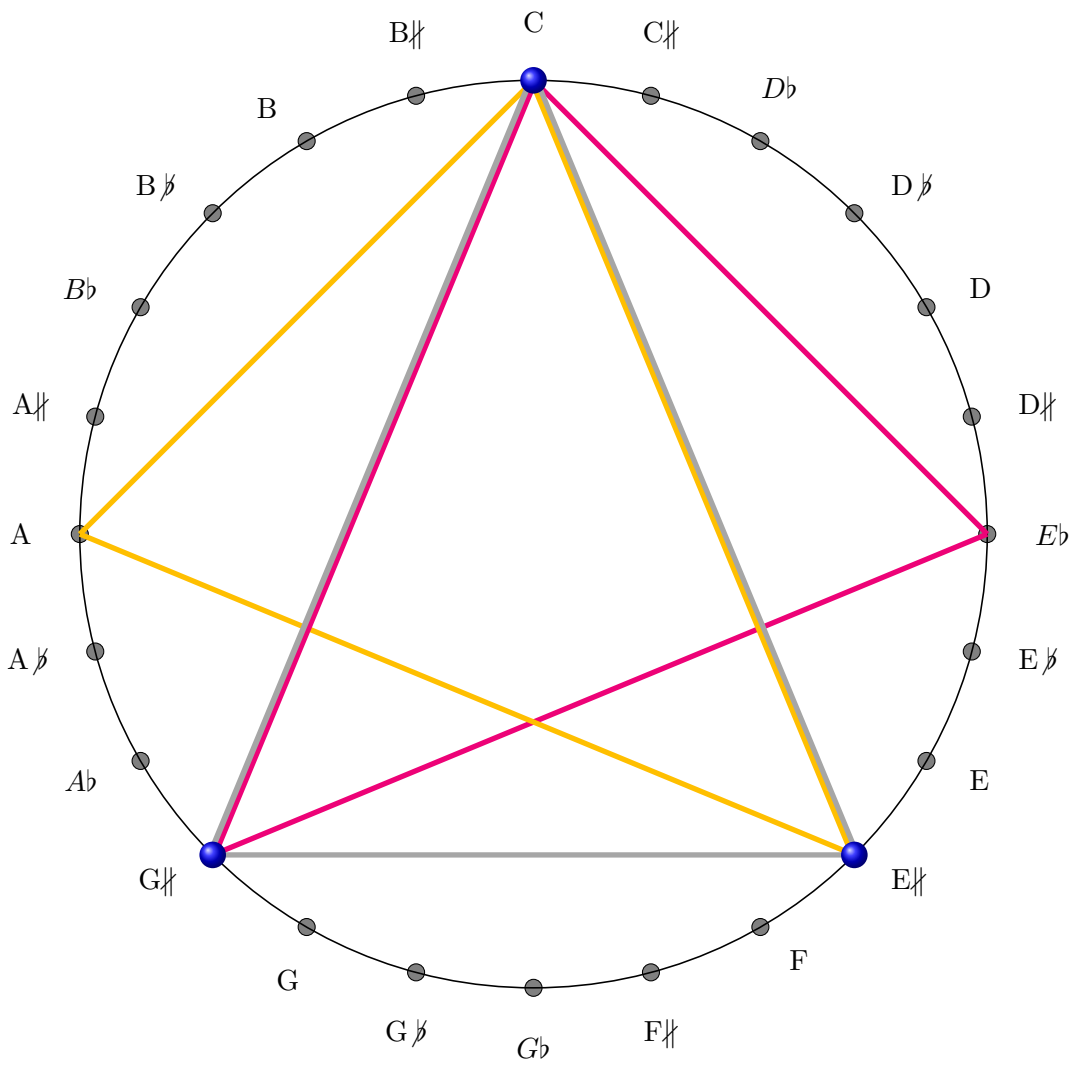

Figure A.25: Triangles $[\frac{18}{2}\ +6]$

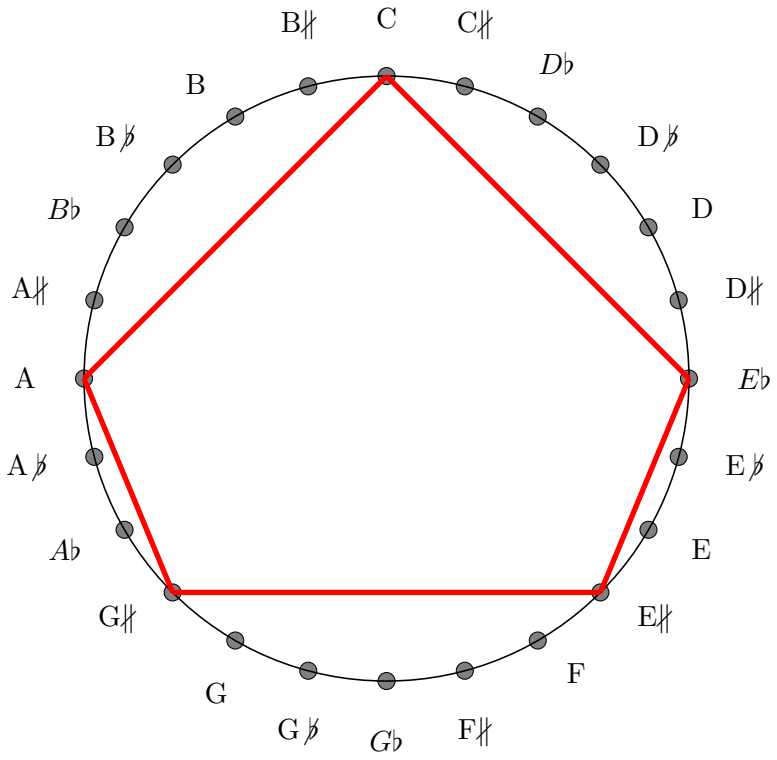

Figure A.26: The Absolute function for $[\frac{18}{2}\ +6]$ triangles

Schema 9:

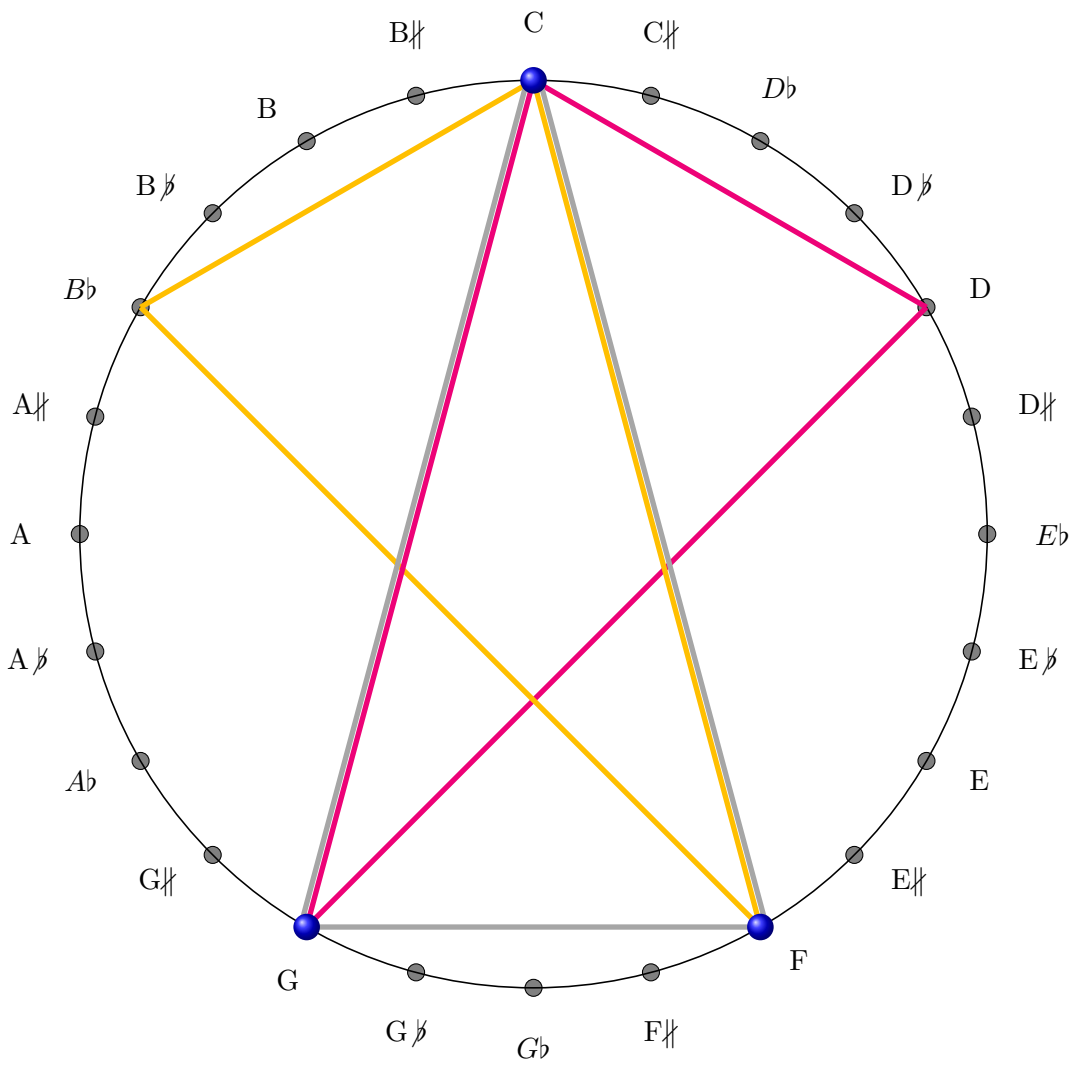

Figure A.27: Triangles $[\frac{20}{2}\ +4]$

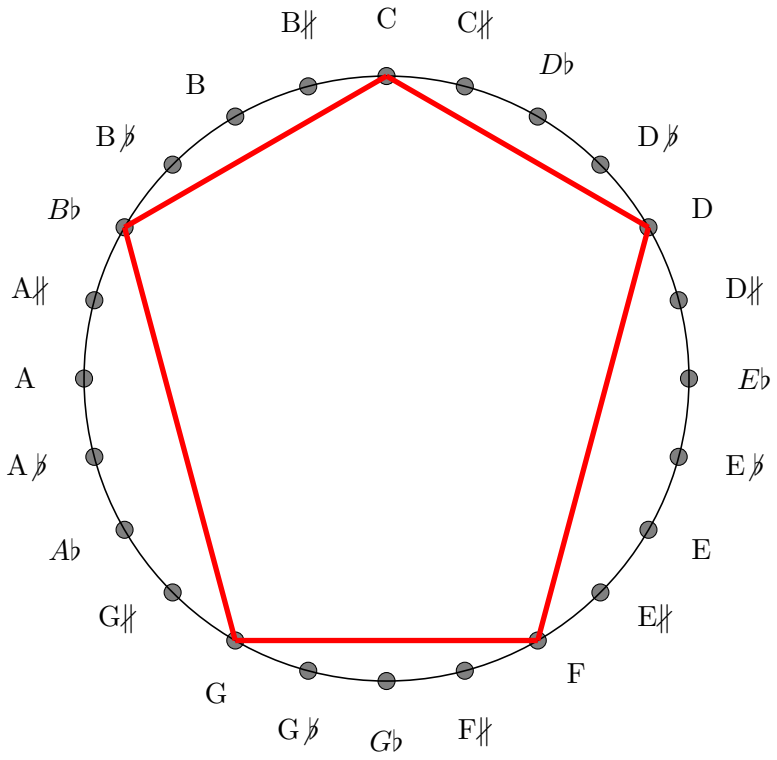

Figure A.28: The Absolute function for $[\frac{20}{2}\ +4]$ triangles

Schema 10:

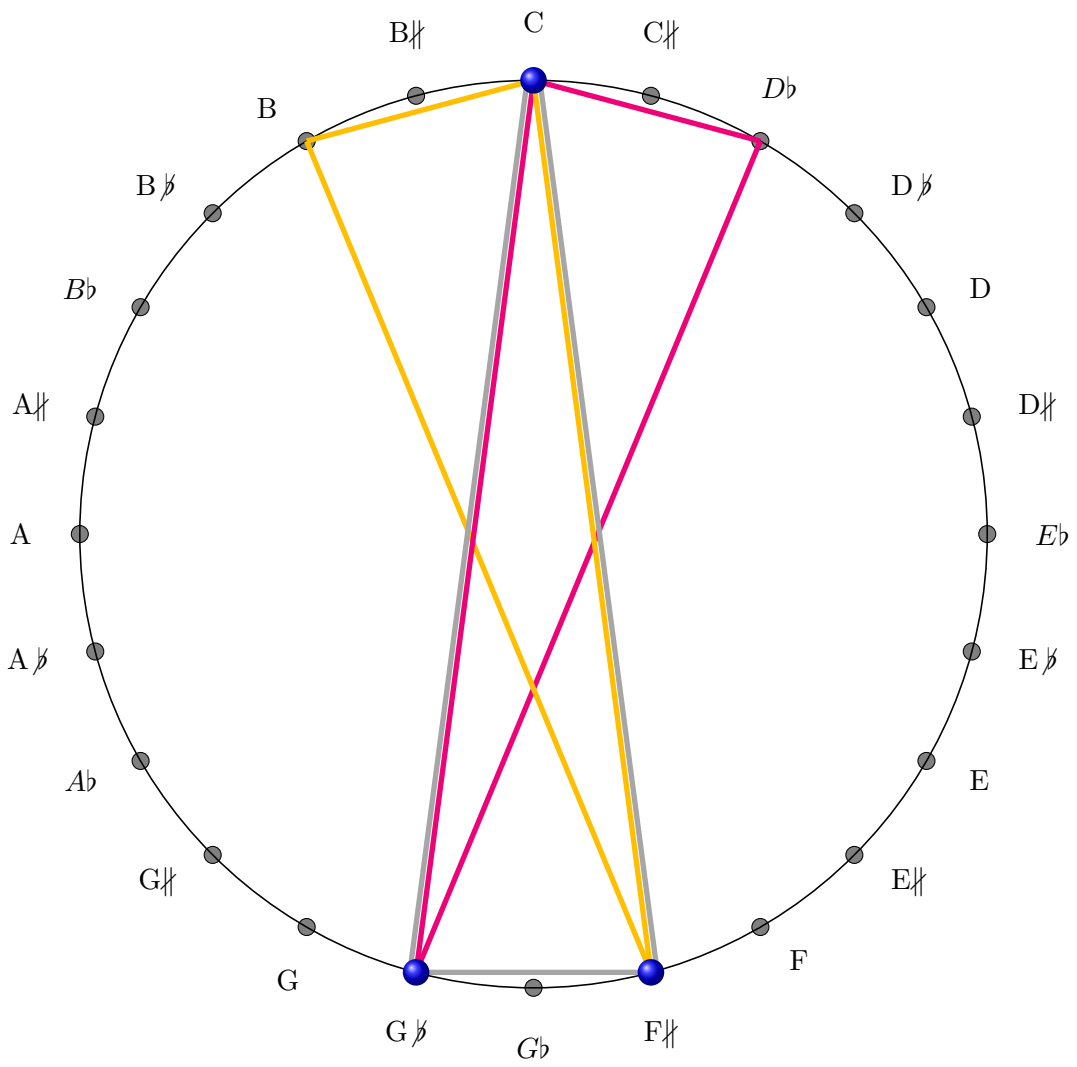

Figure A.29: Triangles $[\frac{22}{2}\ +2]$

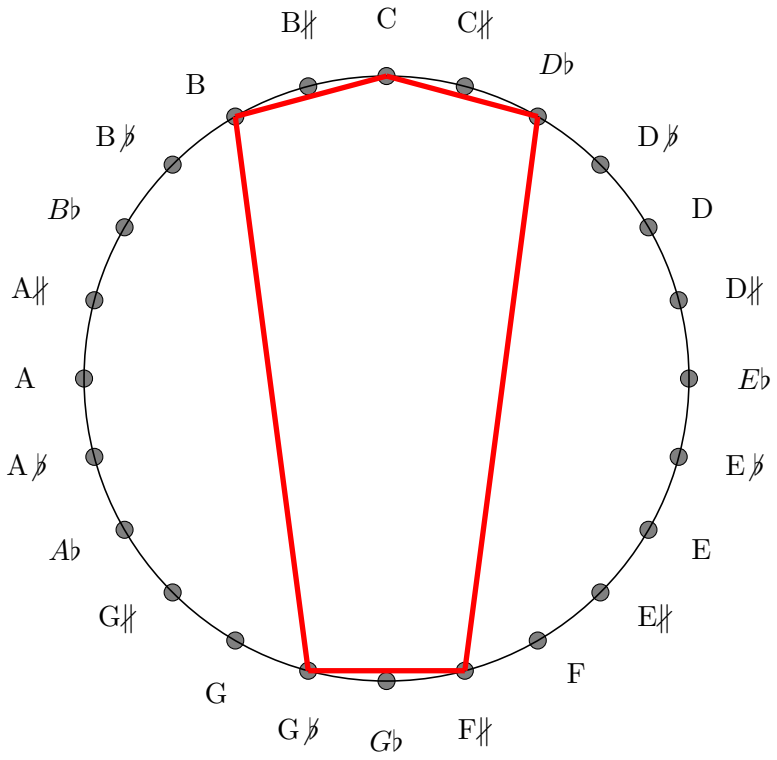

Figure A.30: The Absolute function for $[\frac{22}{2}\ +2]$ triangles

Appendix B

Scale syllabus

Here I list all the scales obtained from the method.

Valter Degani has developed a software which has helped me to compute them all.

The following list is this way organized:

- four sides absolute figures
- triangles absolute figures
- four sides reduced computed figures

The first tabular is therefore presenting all the absolute tonal scales.

Intervals of the scale	**Scale on C**	**Source**	**Number of elements**
♫ [2,11,2,11]	{C;D♭;F♯-;G♭+;B}	[3 sides: 22/2 + 2]	5
♫ [4,6,4,6]	{C;D;F;G;B♭}	[3 sides: 20/2 + 4]	5
♫ [6,3,6,3]	{C;E♭;F-;G♯-;A}	[3 sides: 18/2 + 6]	5
♫ [7,5,4,7]	{C;E♭+;F;G;A♭+}	[3 sides: 14/2 + 10]	5
♫ [5,5,4,5]	{C;D♯-;F;G;A♯-}	[3 sides: 10/2 + 14]	5
♫ [3,3,12,3]	{C;D♭+;E♭;A;B♭+}	[3 sides: 6/2 + 18]	5
♫ [2,2,10,2]	{C;D♭;D;B♭;B}	[3 sides: 4/2 + 20]	5
♫ [1,3,18,1]	{C;C♯-;D♭;B;B♯-}	[3 sides: 2/2 + 22]	5
♫ [1,1,11,1,1,11,1]	{C;C♯-;D♭;F♯-;G♭;G♭+;B;B♯-}]	[4 sides: 1 + 11]	8
♫ [2,2,10,2,2,10,2]	{C;D♭;D;F;G♭;G;B♭;B}	[4 sides: 2 + 10]	8
♫ [3,3,3,9,3,3,3]	{C;D♭+;E♭;E♯-;G♭;G♯-;A;B♭+}	[4 sides: 3 + 9]	8
♫ [5,2,3,4,4,5,3]	{C;D♯-;E♭+;F;G♭;G;A♭+;A♯-}	[4 sides: 5 + 7]	8

Table B.1: Absolute tonal scales.

As it can be observed there have been left out the values:

- 4 side: a+b when a = b;
- 4 sides: a+b when b = 2a;
- 3 sides: a/2+b when a/2 = b;
- 3 sides: a/2+b when a/2 = b/2

Since they represent a special property (equivalence or doubling) they generate special results which can not be considered as scale (such as diminished arpeggio).

It is now presented the tabulars with all the scales obtained computing the reductions with the invariant in 4 sides figures. This values will be excluded also into the reduction process, since they do not generate interesting results.

I now start with all the reduced computation in the couple of value: [1 + 11]:

Intervals of the scale	**Scale on C**	**Number of elements**
♫ [11; 1; 4; 1]	{ C; G♭-; G♭; B; C-}	5
♫ [11; 1; 1; 4; 1]	{ C; G♭-; G♭; F♯+; B; C-}	6
♫ [11; 2; 4; 1]	{ C; G♭-; F♯+; B; C-}	5
♫ [1; 10; 1; 4; 1]	{ C; D♭-; G♭-; G♭; B; C-}	6
♫ [1; 10; 4; 1]	{ C; D♭-; G♭-; B; C-}	5
♫ [1; 10; 1; 1; 4; 1]	{ C; D♭-; G♭-; G♭; F♯+; B; C-}	7
♫ [1; 10; 2; 4; 1]	{ C; D♭-; G♭-; F♯+; B; C-}	6
[1; 1; 9; 2; 4; 1]	{ C; D♭-; D♭; G♭-; F♯+; B; C-}	7
[2; 9; 2; 4; 1]	{ C; D♭; G♭-; F♯+; B; C-}	6
[1; 1; 9; 4; 1]	{ C; D♭-; D♭; G♭-; B; C-}	6
[11; 1; 1; 5]	{ C; G♭-; G♭; F♯+; C-}	5
[1; 10; 1; 5]	{ C; D♭-; G♭-; G♭; C-}	5
[1; 10; 1; 1; 5]	{ C; D♭-; G♭-; G♭; F♯+; C-}	6
[1; 1; 9; 1; 1; 5]	{ C; D♭-; D♭; G♭-; G♭; F♯+; C-}	7
[2; 9; 1; 1; 5]	{ C; D♭; G♭-; G♭; F♯+; C-}	6
[1; 1; 9; 1; 5]	{ C; D♭-; D♭; G♭-; G♭; C-}	6
[11; 1; 1; 4]	{ C; G♭-; G♭; F♯+; B}	5
[1; 10; 1; 1]	{ C; D♭-; G♭-; G♭; F♯+}	5
[1; 1; 9; 1; 1]	{ C; D♭-; D♭; G♭-; G♭; F♯+}	6
[2; 9; 1; 1]	{ C; D♭; G♭-; G♭; F♯+}	5
[1; 11; 4; 1]	{ C; D♭-; G♭; B; C-}	5
[1; 11; 1; 5]	{ C; D♭-; G♭; F♯+; C-}	5
[1; 1; 10; 1; 5]	{ C; D♭-; D♭; G♭; F♯+; C-}	6
[1; 1; 10; 5]	{ C; D♭-; D♭; G♭; C-}	5
[1; 10; 1; 1; 4]	{ C; D♭-; G♭-; G♭; F♯+; B}	6
[1; 11; 1; 4; 1]	{ C; D♭-; G♭; F♯+; B; C-}	6
[1; 1; 10; 1]	{ C; D♭-; D♭; G♭; F♯+}	5
[1; 1; 9; 2; 5]	{ C; D♭-; D♭; G♭-; F♯+; C-}	6
[1; 1; 9; 2; 4]	{ C; D♭-; D♭; G♭-; F♯+; B}	6
[1; 1; 11; 4; 1]	{ C; D♭-; D♭; F♯+; B; C-}	6
[1; 1; 9; 2]	{ C; D♭-; D♭; G♭-; F♯+}	5
[1; 1; 11; 5]	{ C; D♭-; D♭; F♯+; C-}	5

Table B.2: Tonal scales from reduction of [1+11]

I now go on with all the reduced computation in the couple of value: [2 + 10]:

Intervals of the scale	**Scale on C**	**Number of elements**
[10; 2; 2; 2]	{ C; F; G♭; B♭; B}	5
[10; 2; 2; 2; 2]	{ C; F; G♭; G; B♭; B}	6
[10; 4; 2; 2]	{ C; F; G; B♭; B}	5
[2; 8; 2; 2; 2]	{ C; D♭; F; G♭; B♭; B}	6
[2; 8; 2; 2]	{ C; D♭; F; B♭; B}	5
[2; 8; 2; 2; 2; 2]	{ C; D♭; F; G♭; G; B♭; B}	7
[2; 8; 4; 2; 2]	{ C; D♭; F; G; B♭; B}	6
[2; 2; 6; 4; 2; 2]	{ C; D♭; D; F; G; B♭; B}	7
[4; 6; 4; 2; 2]	{ C; D; F; G; B♭; B}	6
[2; 2; 6; 2; 2]	{ C; D♭; D; F; B♭; B}	6
[10; 2; 2; 4]	{ C; F; G♭; G; B}	5
[2; 8; 2; 4]	{ C; D♭; F; G♭; B}	5
[2; 8; 2; 2; 4]	{ C; D♭; F; G♭; G; B}	6
[2; 2; 6; 2; 2; 4]	{ C; D♭; D; F; G♭; G; B}	7
[4; 6; 2; 2; 4]	{ C; D; F; G♭; G; B}	6
[2; 2; 6; 2; 4]	{ C; D♭; D; F; G♭; B}	6
[10; 2; 2; 2]	{ C; F; G♭; G; B♭}	5
[2; 8; 2; 2]	{ C; D♭; F; G♭; G}	5
[2; 2; 6; 2; 2]	{ C; D♭; D; F; G♭; G}	6
[4; 6; 2; 2]	{ C; D; F; G♭; G}	5
[2; 10; 2; 2]	{ C; D♭; G♭; B♭; B}	5
[2; 10; 2; 4]	{ C; D♭; G♭; G; B}	5
[2; 2; 8; 2; 4]	{ C; D♭; D; G♭; G; B}	6
[2; 2; 8; 4]	{ C; D♭; D; G♭; B}	5
[2; 8; 2; 2; 2]	{ C; D♭; F; G♭; G; B♭}	6
[2; 10; 2; 2; 2]	{ C; D♭; G♭; G; B♭; B}	6
[2; 2; 8; 2]	{ C; D♭; D; G♭; G}	5
[2; 2; 6; 4; 4]	{ C; D♭; D; F; G; B}	6
[2; 2; 6; 4; 2]	{ C; D♭; D; F; G; B♭}	6
[2; 2; 10; 2; 2]	{ C; D♭; D; G; B♭; B}	6
[2; 2; 6; 4]	{ C; D♭; D; F; G}	5
[2; 2; 10; 4]	{ C; D♭; D; G; B}	5

Table B.3: Tonal scales from reduction of [2+10]

I now go on with the couple of values: [3 + 9]:

Intervals of the scale	**Scale on C**	**Number of elements**
♫ [9; 3; 0; 3]	{ C; F-; G♭; A; B♭+}	5
♫ [9; 3; 3; 0; 3]	{ C; F-; G♭; A♭-; A; B♭+}	6
♫ [9; 6; 0; 3]	{ C; F-; A♭-; A; B♭+}	5
♫ [3; 6; 3; 0; 3]	{ C; D♭+; F-; G♭; A; B♭+}	6
♫ [3; 6; 0; 3]	{ C; D♭+; F-; A; B♭+}	5
♫ [3; 6; 3; 3; 0; 3]	{ C; D♭+; F-; G♭; A♭-; A; B♭+}	7
[3; 6; 6; 0; 3]	{ C; D♭+; F-; A♭-; A; B♭+}	6
[3; 3; 3; 6; 0; 3]	{ C; D♭+; E♭; F-; A♭-; A; B♭+}	7
[6; 3; 6; 0; 3]	{ C; E♭; F-; A♭-; A; B♭+}	6
[3; 3; 3; 0; 3]	{ C; D♭+; E♭; F-; A; B♭+}	6
[9; 3; 3; 3]	{ C; F-; G♭; A♭-; B♭+}	5
[3; 6; 3; 3]	{ C; D♭+; F-; G♭; B♭+}	5
[3; 6; 3; 3; 3]	{ C; D♭+; F-; G♭; A♭-; B♭+}	6
[3; 3; 3; 3; 3; 3]	{ C; D♭+; E♭; F-; G♭; A♭-; B♭+}	7
[6; 3; 3; 3; 3]	{ C; E♭; F-; G♭; A♭-; B♭+}	6
[3; 3; 3; 3; 3]	{ C; D♭+; E♭; F-; G♭; B♭+}	6
[9; 3; 3; 0]	{ C; F-; G♭; A♭-; A}	5
[3; 6; 3; 3]	{ C; D♭+; F-; G♭; A♭-}	5
[3; 3; 3; 3; 3]	{ C; D♭+; E♭; F-; G♭; A♭-}	6
[6; 3; 3; 3]	{ C; E♭; F-; G♭; A♭-}	5
[3; 9; 0; 3]	{ C; D♭+; G♭; A; B♭+}	5
[3; 9; 3; 3]	{ C; D♭+; G♭; A♭-; B♭+}	5
[3; 3; 6; 3; 3]	{ C; D♭+; E♭; G♭; A♭-; B♭+}	6
[3; 3; 6; 3]	{ C; D♭+; E♭; G♭; B♭+}	5
[3; 6; 3; 3; 0]	{ C; D♭+; F-; G♭; A♭-; A}	6
[3; 9; 3; 0; 3]	{ C; D♭+; G♭; A♭-; A; B♭+}	6
[3; 3; 6; 3]	{ C; D♭+; E♭; G♭; A♭-}	5
[3; 3; 3; 6; 3]	{ C; D♭+; E♭; F-; A♭-; B♭+}	6
[3; 3; 3; 6; 0]	{ C; D♭+; E♭; F-; A♭-; A}	6
[3; 3; 9; 0; 3]	{ C; D♭+; E♭; A♭-; A; B♭+}	6
[3; 3; 3; 6]	{ C; D♭+; E♭; F-; A♭-}	5
[3; 3; 9; 3]	{ C; D♭+; E♭; A♭-; B♭+}	5

Table B.4: Tonal scales from reduction of [3+9]

I now go on with the couple of values [5 + 7]:

Intervals of the scale	**Scale on C**	**Number of elements**
♫ [7; 5; 2; 1]	{ C; E♭+; G♭; G; B♭-}	5
♫ [7; 5; 2; 3; 1]	{ C; E♭+; G♭; G; A♭+; B♭-}	6
♫ [7; 7; 3; 1]	{ C; E♭+; G; A♭+; B♭-}	5
♫ [5; 2; 5; 2; 1]	{ C; E♭-; E♭+; G♭; G; B♭-}	6
♫ [5; 2; 7; 1]	{ C; E♭-; E♭+; G; B♭-}	5
♫ [5; 2; 5; 2; 3; 1]	{ C; E♭-; E♭+; G♭; G; A♭+; B♭-}	7
[5; 2; 7; 3; 1]	{ C; E♭-; E♭+; G; A♭+; B♭-}	6
[5; 2; 3; 4; 3; 1]	{ C; E♭-; E♭+; F; G; A♭+; B♭-}	7
[7; 3; 4; 3; 1]	{ C; E♭+; F; G; A♭+; B♭-}	6
[5; 2; 3; 4; 1]	{ C; E♭-; E♭+; F; G; B♭-}	6
[7; 5; 5; 1]	{ C; E♭+; G♭; A♭+; B♭-}	5
[5; 2; 5; 1]	{ C; E♭-; E♭+; G♭; B♭-}	5
[5; 2; 5; 5; 1]	{ C; E♭-; E♭+; G♭; A♭+; B♭-}	6
[5; 2; 3; 2; 5; 1]	{ C; E♭-; E♭+; F; G♭; A♭+; B♭-}	7
[7; 3; 2; 5; 1]	{ C; E♭+; F; G♭; A♭+; B♭-}	6
[5; 2; 3; 2; 1]	{ C; E♭-; E♭+; F; G♭; B♭-}	6
[7; 5; 2; 3]	{ C; E♭+; G♭; G; A♭+}	5
[5; 2; 5; 5]	{ C; E♭-; E♭+; G♭; A♭+}	5
[5; 2; 3; 2; 5]	{ C; E♭-; E♭+; F; G♭; A♭+}	6
[7; 3; 2; 5]	{ C; E♭+; F; G♭; A♭+}	5
[5; 7; 2; 1]	{ C; E♭-; G♭; G; B♭-}	5
[5; 7; 5; 1]	{ C; E♭-; G♭; A♭+; B♭-}	5
[5; 5; 2; 5; 1]	{ C; E♭-; F; G♭; A♭+; B♭-}	6
[5; 5; 2; 1]	{ C; E♭-; F; G♭; B♭-}	5
[5; 2; 5; 2; 3]	{ C; E♭-; E♭+; G♭; G; A♭+}	6
[5; 7; 2; 3; 1]	{ C; E♭-; G♭; G; A♭+; B♭-}	6
[5; 5; 2; 5]	{ C; E♭-; F; G♭; A♭+}	5
[5; 2; 3; 4; 3]	{ C; E♭-; E♭+; F; G; A♭+}	6
[5; 5; 4; 3; 1]	{ C; E♭-; F; G; A♭+; B♭-}	6
[5; 2; 3; 7; 1]	{ C; E♭-; E♭+; F; A♭+; B♭-}	6
[5; 2; 3; 7]	{ C; E♭-; E♭+; F; A♭+}	5
[5; 5; 7; 1]	{ C; E♭-; F; A♭+; B♭-}	5

Table B.5: Tonal scales from reduction of [5+7]

I now list all the modes of absolute pentatonic coming from triangles with spare constructive values (I skip the pair values since they drive back to half tone constructed scales, so they are not vital to be listed here).

Intervals of the scale (root based)	**Scale on C**	**Source**
♫ [R; 3♭-; 4; 7♭-]	{ C; E♭-; F; G; B♭-}	10/2 + 14
♫ [R; 3♭-; 4-; 5; 7♭-]	{ C; E♭-; F-; G; B♭-}	10/2 + 14
♫ [R; 2; 4-; 5; 7♭-]	{ C; D; F-; G; B♭-}	10/2 + 14
♫ [R; 3♭-; 4; 6♭-; 7♭]	{ C; E♭-; F; A♭-; B♭}	10/2 + 14
♫ [R; 3♭-; 4; 6♭-; 7♭-]	{ C; E♭-; F;A♭-; B♭-}	10/2 + 14
♫ [R; 3♭-; 4; 5; 6-]	{ C; E♭-; F; G; A-}	14/2 + 10
♫ [R; 3♭-; 4-; 4; 6-]	{ C; E♭-; F-; F; A-}	14/2 + 10
♫ [R; 2; 3-; 5; 7♭-]	{ C; D; E-; G; B♭-}	14/2 + 10
♫ [R; 2-; 4; 6-; 7♭]	{ C; D-; F; A-; B♭}	14/2 + 10
♫ [R; 4-; 5; 6-; 7-]	{ C; F-; G; A-; B-}	14/2 + 10
♫ [R; 2-; 3♭; 4-; 5♭]	{ C; D-; E♭; F-; G♭}	6/2 + 18
♫ [R; 2-; 3♭; 4-; 7-]	{ C; D-; E♭; F-; B-}	6/2 + 18
♫ [R; 2-; 3♭; 6; 7-]	{ C; D-; E♭; A; B-}	6/2 + 18
♫ [R; 3♭-; 6♭-; 6; 7-]	{ C; E♭-; A♭-; A; B-}	6/2 + 18
♫ [R; 5♭; 6♭-; 6; 7-]	{ C; G♭; A♭-; A; B-}	6/2 + 18
♫ [R; 3♭; 4-; 6♭-; 6]	{ C; E♭; F-; A♭-; A}	18/2 + 6
♫ [R; 2♭; 4-; 5♭; 6]	{ C; D♭; F-; G♭; A}	18/2 + 6
♫ [R; 3♭; 3-; 6♭-; 7-]	{ C; E♭; E-; A♭-; B-}	18/2 + 6
♫ [R; 2-; 4-; 6♭-; 6]	{ C; D-; F-; A♭-; A}	18/2 + 6
♫ [R; 3♭; 5♭; 6♭-; 7-]	{ C; E♭; G♭; A♭-; B-}	18/2 + 6
[R; 2♭; 2; 5-; 6♭]	{ C; D♭; D; G-; A♭-}	22/2 + 2
[R; 2♭; 5♭-; 5-; 7]	{ C; D♭; G♭-; G-; B}	22/2 + 2
♫ [R; 4-; 5♭-; 7♭; 7]	{ C; F-; G♭-; B♭; B}	22/2 + 2
♫ [R; 2♭; 5♭-; 5-; 6-]	{ C; D♭; G♭-; G- ; A-}	22/2 + 2
[R; 4-; 5♭-; 6-; 7]	{ C; F-; G♭-; A-; B}	22/2 + 2

Table B.6: Absolute pentatonic scales from triangles.

I now list all the pentatonic from triangles re-organized by tonal area re-deduction. I proposes them in another order and furthermore I here propose their diatonic develop, as explained in chapter 5.5. (I skip the pair values since they drive back to half tone constructed scales, so they are not vital to be listed here).

Intervals of the scale	**C Scale**	**Tonal ambit**	**Diatonic construct**
♫ [R; 4-; 5; 6-; 7-]	{ C; F-; G; A-; B-}	MajorScale	{C; II; III; F-; G; A-; B-}
♫ [R; 3♭-; 6♭-; 6; 7♭-]	{ C; E♭-; A♭-; A; B♭-}	MinorScale	{C; II; E♭-; IV; V; A♭-; B♭-}
♫ [R; 3♭-; 4; 5; 7♭-]	{ C; E♭-; F; G; B♭-}	MinorScale	{C; II; E♭-; F; G; VI; B♭-}
♫ [R; 3♭-; 4-; 5; 7♭-]	{ C; E♭-; F-; G; B♭-}	MinorScale	{C; II; E♭-; F-; G; VI; B♭-}
♫ [R; 3♭-; 4; 6♭-; 7♭]	{ C; E♭-; F; A♭-; B♭}	MinorScale	{C; II; E♭-; F; V; A♭-; B♭}
[R; 3♭-; 4; 6♭-; 7♭-]	{ C; E♭-; F; A♭-; B♭-}	MinorScale	{C; II; E♭-; F; V; A♭-; B♭-}
♫ [R; 3♭-; 4-; 4; 6-]	{ C; E♭-; F-; F; A-}	MinorScale	{C; II; E♭-; F-; F; A-; VII}
♫ [R; 2-; 3♭; 6; 7-]	{ C; D-; E♭-; A; B-}	MinorScale	{C; D-; E♭-; IV; V; A; B-}
♫ [R; 2-; 4-; 5; 7♭-]	{ C; D-; F-; G; B♭-}	MinorScale	{C; D-; III; F-; G; VI; B♭-}
♫ [R; 2; 4-; 5; 7♭-]	{ C; D; F-; G; B♭-}	MinorScale/Sus	{C; D; III; F-; G; VI; B♭-}
♫ [R; 2-; 4; 6-; 7♭]	{ C; D-; F; A-; B♭}	MinorScale/Sus	{C; D-; III; F; V; A-; B♭}
♫ [R; 2; 3-; 5; 7♭-]	{ C; D; E-; G; B♭-}	Mixolidyan	{C; D; E-; IV; G; VI; B♭-}
[R; 5♭; 6♭-; 6; 7-]	{ C; G♭; A♭-; A; B-}	Diminished	{C; III; IV; G♭; A♭-; A; B-}
[R; 3♭; 4-; 6♭-; 6]	{ C; E♭; F-; A♭-; A}	Diminished	{C; II; E♭; F-; V; A♭-; A; VII}
[R; 3♭; 6♭-; 6; 7-]	{ C; G♭; A♭-; A; B-}	Diminished	{C; II; III; IV; G♭; A♭-; A; B-}
[R; 3♭; 4-; 6♭-; 6]	{ C; E♭; F-; A♭-; A}	Diminished	{C; II; E♭; F-; V; A♭-; A; VII}
[R; 3♭; 5♭; 6♭-; 7♭-]	{ C; E♭; G♭; A♭-; B♭-}	Diminished	{C; II; E♭; IV; G♭; A♭-; B♭-}
[R; 2-; 3♭; 4-; 5♭]	{ C; D-; E♭; F-; G♭}	Diminished	{C; D-; E♭; F-; G♭; A♭-; A; VII}
[R; 2-; 4-; 6♭-; 6]	{ C; D-; F-; A♭-; A}	Diminished	{C; D-; III; F-; V; VI; VII}
[R; 3♭; 5♭; 6♭-; 7-]	{ C; E♭; G♭; A♭-; B-}	18/2 + 6(?)	?
[R; 2♭; 2; 5-; 6♭]	{ C; D♭; D; G-; A♭-}	22/2 + 2(?)	?
[R; 2♭; 5♭-; 5-; 7]	{ C; D♭; G♭-; G-; B}	22/2 + 2(?)	?
[R; 4-; 5♭-; 7♭; 7]	{ C; F-; G♭-; B♭; B}	22/2 + 2(?)	?
[R; 2♭; 5♭-; 5-; 6-]	{ C; D♭; G♭-; G- ; A-}	22/2 + 2(?)	?
[R; 4-; 5♭-; 6-; 7]	{ C; F-; G♭-; A-; B}	22/2 + 2(?)	?

Table B.7: Pentatonic scales from triangles reorganized by tonal area re-deduction

As it can be observed, the value [22/2 + 2] produces too close notes, therefore it is not valid to generate tonal scales.

Thus also the value [1 +11] will be skipped in the next tabular.

I now list all the pentatonic from rectangles re-organized by tonal area re-deduction.

Intervals of the scale	C Scale	Tonal ambit	Diatonic construct
[R; 4-; 5♭; 6; 7♭+]	{C; F-; G♭; A; B♭+}	Mixolydian5♭	{C; II; III; F-; G♭; A; B♭+}
♫ [R; 4-; 6♭-; 6; 7♭+]	{C; F-; A♭-; A; B♭+}	Mixolydian5♯	{C; II; III; F-; A♭-; A; B♭+}
[R; 2♭+; 4-; 6; 7♭+]	{C; D♭+; F-; A; B♭+}	Phrigian	{C; D♭+; III; F-; V; VI; B♭+}
♫ [R; 4-; 5♭; 6♭-; 7♭+]	{C; F-; G♭; A♭-; B♭+}	Locrian	{C; II; III; F-; G♭; A♭-; B♭+}
[R; 2♭+; 4; 5♭; 7♭+]	{C; D♭+; F-; G♭; B♭+}	Locrian	{C; D♭+; III; F-; G♭; VI; B♭+}
♫ [R; 4; 5♭; 6♭-; 6]	{C; F-; G♭; A♭-; A}	Mixolydian5♭/6♭	{C; II; III; F-; G♭; A♭-; A; VII}
[R; 2♭+; 4-; 5♭; 6♭-]	{C; D♭+; F-; G♭; A♭-}	Locrian	{C; D♭+; III; F-; G♭; A♭-; VII}
[R; 3♭; 4-; 5♭; 6♭-]	{C; E♭; F-; G♭; A♭-}	Locrian	{C; II; E♭; F-; G♭; A♭-; VII}
[R; 2♭+; 5♭; 6; 7♭+]	{C; D♭+; G♭; A; B♭+}	Superlocrian	{C; D♭+; III; IV; G♭; A; B♭+}
[R; 2♭+; 5♭; 6♭-; 7♭+]	{C; D♭+; G♭; A♭-; B♭+}	Locrian	{C; D♭+; III; IV; G♭; A♭-; B♭+}
[R; 2♭+; 3♭; 5♭; 6♭-]	{C; D♭+; E♭; G♭; A♭-}	Locrian	{C; D♭+; E♭; IV; G♭; A♭-; VII}
[R; 2♭+; 3♭; 5♭; 6♭-]	{C; D♭+; E♭; F-; A♭-}	Locrian	{C; D♭+; E♭; F-; V; A♭-; VII}
[R; 2♭+; 3♭; 6♭-; 7♭+]	{C; D♭+; E♭; A♭-; B♭+}	Locrian	{C; D♭+; E♭; IV; V; A♭-; B♭+}
[R; 3♭; 5♭; 5; 7♭-]	{C; E♭+; G♭; G; B♭-}	LydianDominant /♭9	{C; II; E♭; G♭; G; VI; B♭-}
[R; 3♭; 5; 6♭+; 7♭-]	{C; E♭+; G; A♭+; B♭-}	Aeolian	{C; II; E♭+; IV; G; A♭+; B♭-}
[R; 3♭; 3♭+; 5; 7♭-]	{C; E♭-; E♭+; G; B♭-}	Blues	{C; E♭-; E♭+; IV; G; VI; B♭}
[R; 3♭; 5♭; 6♭+; 7♭-]	{C; E♭+; G♭; A♭+; B♭-}	Locrian	{C; II; E♭; IV; G♭; A♭+; B♭-}
[R; 3♭-; 3♭+; 5♭; 7♭-]	{C; E♭-; E♭+; G♭; B♭-}	Blues	{C; E♭-; E♭+; G♭; VI; B♭-}
[R; 3♭+; 5♭; 5; 6♭+]	{C; E♭+; G♭; G; A♭+}	LydianDominant /♭9	{C; II; E♭+; G♭; G; A♭+; VII}
[R; 3♭-; 3♭+; 5♭; 6♭+]	{C; E♭-; E♭+; G♭; A♭+}	Blues	{C; E♭-; E♭+; G♭; V; A♭+; VII}
[R; 3♭+; 4; 5♭; 6♭+]	{C; E♭+; F; G♭; A♭+}	Locrian	{C; II; E♭+; F; G♭; V; A♭+; VII}
[R; 3♭-; 5♭; 5; 7♭-]	{C; E♭-; G♭; G; B♭-}	LydianDominant	{C; II; E♭-; G♭; G; VI; B♭-}
[R; 3♭-; 5♭; 6♭+; 7♭-]	{C; E♭-; G♭; A♭+; B♭-}	Locrian	{C; II; E♭-; G♭; V; A♭+; B♭-}
[R; 3♭-; 4; 5♭; 7♭-]	{C; E♭-; F; G♭ ; B♭-}	Locrian	{C; II; E♭-; F; G♭; VI; B♭-}
[R; 3♭-; 4; 5♭-; 6♭+]	{C; E♭-; F; G♭; A♭+}	Locrian	{C; II; E♭-; F; G♭; A♭+; VII}
[R; 3♭-; 3♭+; 4; 6♭+]	{C; E♭-; E♭+; F; A♭+}	Superlocrian	{C; II; E♭-; E♭+; F; V; A♭+; VII}
[R; 3♭-; 4; 6♭+; 7♭-]	{C; E♭-; F; A♭+; B♭-}	Locrian	{C; II; E♭-; F; V; A♭+; B♭-}

Table B.8: Pentatonic scales from rectangles reorganized by tonal area re-deduction

I now list a summa of all the gotten scales the system has provided:

- 4 eight notes scales
- 12 seven notes scales
- 88 six notes scales
- 106 five notes scales

Finally the system has provided 210 valid scales.

Appendix C

Hints on microtonality

Here I just want to hint about the many other researches that has been made upon microtonality.

Intervals other than the half step are being used in several music cultures. In arabic music there are a lot of intervals others from the half step. More precisely in the Arabic Maquam there is plenty of scales which make use of quarter tones. The same in the Draghastan of Persian music and in South-east Asia (Javanese gamelan music for instance).

In European classical music quarter tones has been approached in the beginning of XX century. Composers such as Alois Hába, Charles Ives and Ivan Aleksandrovic Vysnegradskij has composed using quarter tones, see, as example:

- Alois Hába. *Sonata for Quarter tone Piano op.62.* , 1946-1947. [1]
- Ivan A. Wyschnegradsky. *24 Préludes dans tous les tons de l'échelle chromatique diatonisée à 13 sons, pour 2 pianos en quarts de ton, Op. 22*, 1934, révision 1960. [2]
- Charles Ives. *Three Quarter-Tone Pieces*, 1903-1923. [3]

Later on the French structuralist school has researched about new system for organizing music which take in consideration pitch disjointed from the musical notes. We can find microtones in Penderecki, in Harry Partch and others.

Concerning the rock music, experiments on quarter tones has been made. The Australian band King Gizzard And The Lizard Wizard has released in 2017 *Flying Microtonal Banana*, an album which has been played only with microtonal instruments. [4]

[1] Online video available at: `https://www.youtube.com/watch?v=s7vZURdhucM` [Accessed 5 March 2016].
[2] Online video available at: `https://www.youtube.com/watch?v=tDroa5WTU34` [Accessed 9 sept. 2016].
[3] Online videos available at:

for 1st piece: `https://www.youtube.com/watch?v=izFgt2tZOOc` [Accessed 6 Oct. 2016].

for 2nd piece: `https://www.youtube.com/watch?v=5ZExi38G9AE` [Accessed 8 Oct. 2016].

for 3rd piece: `https://www.youtube.com/watch?v=AkxLliTkxTQ` [Accessed 12 Oct. 2016].

[4] Online video available at: `https://www.youtube.com/watch?v=D0BsgJxw208` [Accessed 10 Feb. 2017].

Furthermore I suggest the Youtube channel of Tolgahan Cogulu, whose has undertaken a research project on quarter tone using classical guitar which I think it is really inspiring: {https://www.youtube.com/channel/UCf7D886oBahxSSwBRVIib0A}.

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